1. (a) (i) $2a + b$  
   correct expression for $2a + b$  
   $\text{eg} \quad \left( \frac{3}{2}, 5, -2, 5, -2 \right)$  
   (A1)  
   (ii) correct substitution into length formula  
   $\text{eg} \quad \sqrt{5^2 + 2^2}, \sqrt{5^2 + 2^2}$  
   (A1)  
   [4 marks]  

(b) valid approach  
   $\text{eg} \quad -\left(2a + b\right), \ 5 + x = 0, -2 + y = 0$  
   $\epsilon = \left( -5, 2 \right)$  
   (A1)  
   [2 marks]  

Total [6 marks]  

2. (a) $x = 1, \ y = -3$ (accept $(1,0), (-3,0)$)  
   (A1A1)  
   [2 marks]  

(b) METHOD 1  
   attempt to find $x$-coordinate  
   $\text{eg} \ 1 + 1 = x, -\frac{b}{2a}, f'(x) = 0$  
   correct value, $x = -1$ (may be seen as a coordinate in the answer)  
   (M1)  
   attempt to find their $y$-coordinate  
   $\text{eg} \quad f(-1), -2 + 2, y = \frac{-b}{4a}$  
   $y = -4$  
   vertex $(-1, -4)$  
   (A1)  
   [4 marks]  

METHOD 2  
   attempt to complete the square  
   $\text{eg} \quad x^2 + 2x + 1 - 1 - 3$  
   attempt to put into vertex form  
   $\text{eg} \quad (x + 1)^2 - 4, (x - 1)^2 + 4$  
   vertex $(-1, -4)$  
   (A1A1)  
   [4 marks]  

Total [6 marks]  

3. (a) evidence of choosing product rule  
   $\text{eg} \quad \text{uv}' + vu'$  
   correct derivatives (must be seen in the product rule) $\cos x, 2x$  
   $f'(x) = x' \cos x + 2x \sin x$  
   (A1)  
   [4 marks]  

(b) substituting $\frac{\pi}{2}$ into their $f'(x)$  
   $\text{eg} \quad f'= \left( \frac{\pi}{2} \right), \left( \frac{\pi}{2} \right) \cos \left( \frac{\pi}{2} \right) + 2 \left( \frac{\pi}{2} \right) \sin \left( \frac{\pi}{2} \right)$  
   correct values for both $\sin \frac{\pi}{2}$ and $\cos \frac{\pi}{2}$ seen in $f'(x)$  
   (A1)  
   [3 marks]  

Total [7 marks]
4. (a) attempt to solve for $X$

e.g. $X = C - B, X + B = CA^{-1}, A^{-1}(C - B), A^{-1}C - B$

$$X = (C - B)A^{-1} = (CA^{-1} - BA^{-1})$$

A1 N2

(b) METHOD 1

$$C - B = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$$

A1

correct substitution into formula for $2 \times 2$ inverse

e.g. $A^{-1} = \frac{1}{4} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}$

A1

attempt to multiply $(C - B)$ and $A^{-1}$ (in any order)

e.g. $\begin{pmatrix} -2 & 3 \\ 8 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 4 & -6 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ 8 & 8 \end{pmatrix}$

two correct elements

$$X = \begin{pmatrix} 1 & 0 \\ 11 & -5 \end{pmatrix}$$

A2 N3

Note: Award A1 for three correct elements.

METHOD 2

correct substitution into formula for $2 \times 2$ inverse

e.g. $A^{-1} = \frac{1}{4} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}$

A1

attempt to multiply either $RA^{-1}$ or $CA^{-1}$ (in any order)

e.g. $\begin{pmatrix} 0 & -3 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 6 \\ 4 & 2 \end{pmatrix}$

two correct entries

one correct multiplication

e.g. $\begin{pmatrix} -3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 7 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 2 & -5 \end{pmatrix}$

$$X = \begin{pmatrix} 1 & 0 \\ 11 & -5 \end{pmatrix}$$

A2 N3

Note: Award A1 for three correct elements.

5. (a) METHOD 1

attempt to set up equation

e.g. $2 = \sqrt{5} - 5, 2 = \sqrt{5} - 5$

A1

correct working

e.g. $4 - y = 5, x = x^2 + 5$

$g'(2) = 9$

A1 N2

METHOD 2

interchanging $x$ and $y$ (seen anywhere)

e.g. $x = \sqrt{5} - 5$

A1

correct working

e.g. $x^2 = y - 5, y = x^2 + 5$

$g'(2) = 9$

A1 N2

(b) recognizing $g'(3) = 30$

e.g. $f(30)$

A1

correct working

e.g. $(f + g'(3)) = (30) = \sqrt{5}$

$g'(2) = 9$

A1 N2

Note: Award A0 for multiple values, e.g. $5 \pm 5$

Total [6 marks]
6. attempt to integrate which involves ln
eg \( \ln(2x - 5), 12 \ln 2x - 5, \ln 2x \)
correct expression (accept absence of \( C \))
eg \( 12 \ln(2x - 5) \frac{1}{2} + C, \; 6 \ln(2x - 5) \)

attempt to substitute \((4, 0)\) into their integrated \( f \)
eg \( 0 - 6 \ln(2x - 5), 0 - 6 \ln(8 - 5) + C \)
\( C = -6 \ln 3 \)

\( f(x) = 6 \ln(2x - 5) - 6 \ln 3 \left( -6 \ln \left( \frac{2x - 5}{3} \right) \right) \) (accept \( 6 \ln(2x - 5) - \ln 3 \))

\( \text{Note: Except to the FF rule. Allow full FF on incorrect integration which must involve ln.} \)

Total \([6 \text{ marks}]\)

7. \(a\) evidence of correct formula
eg \( \log a - \log b = \log \frac{a}{b}, \; \log 8 + \log 5 = \log 40 \)

\( \text{Note: Ignore missing or incorrect base.} \)

correct working
eg \( \log 8, \; 2^8 = 8 \)
\( \log 40 - \log 5 = 3 \)

\( \text{Total \([6 \text{ marks}]\)} \)

\(b\) attempt to write 8 as a power of 2
eg \( (2^3)^{10}, \; 2^8 = 8, \; 2^x \)
multiplying powers
eg \( 2^{2x + 1}, \; a \log_2 5 \)
correct working
eg \( 2^{2x + 1}, \; \log_2 5^3, \; \left( 2^{2x + 1} \right) \)
\( 8^{\frac{3}{2}} = 125 \)

\( \text{Total \([7 \text{ marks}]\)} \)

SECTION B

8. \(a\) \(i\) valid approach
eg \( \begin{pmatrix} -4 \end{pmatrix}, \; A - B, \; \overrightarrow{\text{AB}} = \overrightarrow{\text{AO}} + \overrightarrow{\text{OB}} \)

\( \overrightarrow{\text{AB}} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} \)

\( \text{A1} \; \text{N2} \)

\( \text{ii} \) any correct equation in the form \( r = a + tb \) (accept any parameter for \( t \))
where \( a = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \) and \( b \) is a scalar multiple of \( \overrightarrow{\text{AB}} \)

eg \( t = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}, \; (x, y, z) = (1, -2, 3) + t(3, -1, 2), \; r = \begin{pmatrix} 1 + 6t \\ 3 - 2t \\ 4t \end{pmatrix} \)

\( \text{Note: Award A1 for } a + tb, \; A1 \text{ for } L_a = a + tb, \; \text{A0 for } r = b + ta. \)

\( \text{[4 marks]} \)

\(b\) recognizing that scalar product \( = 0 \) (seen anywhere)
\( \text{R1} \)
correct calculation of scalar product
eg \( 6(3) - 2(-3) + 4p, \; 18 + 6 + 4p \)
correct working
eg \( 24 + 4p = 0, \; 4p = -24 \)
\( p = -6 \)

\( \text{AG} \; \text{N0} \)

\( \text{[3 marks]} \)

continued ...
Question 8 continued

(c) setting lines equal

\[ L_1 = L_2 \]

\[
\begin{pmatrix}
1 & 1 & 6 \\
3 & 4 & 15 \\
2 & 2 & -6
\end{pmatrix}
\]

eg \[1 \times 2 = -2 \times 2 + 3 \times -3\]

any two correct equations with different parameters

eg \[1 \times 4 = -1 \times 3r, \ -2 \times 2 = 2 - 3r, \ 3 \times 5 = 15 - 6s\]

attempt to solve their simultaneous equations

one correct parameter

eg \[r = \frac{1}{2}, \ s = \frac{5}{3}\]

attempt to substitute parameter into vector equation

eg \[
\begin{pmatrix}
1 & -2 \\
3 & 2 \\
2 & -2
\end{pmatrix}
\begin{pmatrix}
r \\
s
\end{pmatrix} =
\begin{pmatrix}
1 & 4 \\
6 & 1
\end{pmatrix}
\]

\[x = 4\] (accept \((-3, 5)\), ignore incorrect values for \(y\) and \(z\))

9. (a) (i) attempt to find \(P(\text{red}) \times P(\text{red})\)

\[
\begin{pmatrix}
3 & 2 \\
3 & 3 \\
2 & 2
\end{pmatrix}
\]

\[P(\text{none green}) = \frac{6}{56} \begin{pmatrix}
-3 \\
-25
\end{pmatrix}\]

(a) (ii) attempt to find \(P(\text{red}) \times P(\text{green})\)

\[
\begin{pmatrix}
5 & 3 \\
7 & 5 \\
8 & 15
\end{pmatrix}
\]

recognizing two ways to get one red, one green

eg \[2P(R) \times P(G) \times \begin{pmatrix}
5 & 3 \\
7 & 7 \\
8 & 15
\end{pmatrix} + 2\]

\[P(\text{exactly one green}) = \frac{30}{56} \begin{pmatrix}
-15 \\
-28
\end{pmatrix}\]

(b) \(P(\text{both green}) = \frac{20}{56}\) (seen anywhere)

\[P(\text{both green}) = \begin{pmatrix}
\frac{5}{8} \\
\frac{3}{8}
\end{pmatrix}\]

\[E(X) = 0 \times \frac{5}{8} + \frac{30}{56} \times \frac{5}{8} + \frac{20}{56} \times \frac{3}{8}
\]

\[E(X) = \frac{70}{56} \begin{pmatrix}
-5 \\
-7
\end{pmatrix}\]

expected number of green marbles is \(\frac{70}{56} \begin{pmatrix}
-5 \\
-7
\end{pmatrix}\)
Question 9 continued

(c) (i) \( P(\text{jar } B) = \frac{\frac{1}{2}}{3} \cdot \left( \frac{\frac{2}{2}}{3} \right) \)

\( A1 \) \( N1 \)

(ii) \( P(\text{red}|\text{jar } B) = \frac{\frac{6}{8}}{\frac{1}{2}} \cdot \left( \frac{\frac{1}{4}}{3} \right) \)

\( A1 \) \( N1 \)

[2 marks]

(d) recognizing conditional probability

\( eg \ P(A|R), \ P(\text{jar } A \text{ and red}) \)

\( P(\text{red}) \), tree diagram

attempt to multiply along either branch (may be seen on diagram)

\( eg \ P(\text{jar } A \text{ and red}) = \frac{\frac{1}{3}}{5} \cdot \left( \frac{\frac{2}{2}}{3} \right) \)

attempt to multiply along other branch

\( eg \ P(\text{jar } B \text{ and red}) = \frac{\frac{6}{8}}{\frac{1}{2}} \cdot \left( \frac{\frac{1}{4}}{3} \right) \)

adding the probabilities of two mutually exclusive paths

\( eg \ P(\text{red}) = \frac{\frac{1}{3}}{5} \cdot \frac{\frac{2}{2}}{3} + \frac{\frac{6}{8}}{\frac{1}{2}} \cdot \left( \frac{\frac{1}{4}}{3} \right) \)

\( A1 \)

correct substitution

\( eg \ P(\text{jar } A|\text{red}) = \frac{\frac{1}{3}}{5} \cdot \frac{\frac{1}{2}}{3} + \frac{\frac{6}{8}}{\frac{1}{2}} \cdot \left( \frac{\frac{1}{4}}{3} \right) \)

\( A1 \)

\( P(\text{jar } A|\text{red}) = \frac{1}{3} \)

\( A1 \) \( N3 \)

[6 marks]

Total [16 marks]
Question 10 continued

(d) \[ A_{1}A_{1}A_{1} \]

Notes:
Award A1 for shape concave up left of POI and concave down right of POI.
A1 for curve through (0, 0), A1 for increasing throughout.
Sketch need not be drawn to scale. Only essential features need to be clear.

[3 marks]
Total [15 marks]

SECTION A

QUESTION 1

(a) (i) \[ AB = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \] (A2) N2

(ii) \[ A^{-1} = \begin{pmatrix} 1 & -1 \\ -6 & 5 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \] (A1) N1

(b) METHOD 1

\[ A^{-1}C = \begin{pmatrix} 1 & -1 \\ -6 & 5 \end{pmatrix} \begin{pmatrix} 1 & 8 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ -17 \end{pmatrix} \] (A1) N3

METHOD 2

\[ 5x + y = 8 \] (M1)
\[ 6x + 2y = -4 \]

for work towards solving their system
\[ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -17 \end{pmatrix} \] (A1) N3

[7 marks]

QUESTION 2

(a) \[ P(A) = \frac{1}{11} \] (A1) N1

(b) \[ P(B|A) = \frac{2}{10} \] (A2) N2

(c) recognising that \[ P(A\cup B) = P(A) \times P(B|A) \] (M1)
correct values (A1)
\[ e.g. P(A\cup B) = \frac{1}{11} \times \frac{2}{10} \]
\[ P(A\cup B) = \frac{2}{110} \] (A1) N3

[6 marks]
QUESTION 3

evidence of choosing the product rule
\( f'(x) = e^x \times (-\sin x) + \cos x \times e^x \) \( (\text{M1}) \)

substituting \( \pi \)
\( e.g. \quad f'(x) = e^\pi \cos x - e^\pi \sin x, \quad e^\pi (\pi - 1) = -e^\pi \) \( (\text{M1}) \)

taking negative reciprocal
\( e.g. \quad \frac{1}{f'(x)} \) \( (\text{M1}) \)

gradient is \( \frac{1}{e^\pi} \) \( A1 \) \( N3 \)

[6 marks]

QUESTION 4

(a)

<table>
<thead>
<tr>
<th>Function</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>displacement ( \Delta )</td>
<td>( A )</td>
</tr>
<tr>
<td>acceleration ( \dot{A} )</td>
<td>( B )</td>
</tr>
</tbody>
</table>

\( A2A2 \) \( N4 \)

(b) \( t = 3 \)

\( A2 \) \( N2 \)

[6 marks]

QUESTION 5

(a) in any order
translated 1 unit to the right
stretched vertically by factor 2

\( \text{METHOD 1} \)

Finding coordinates of image on \( g \)
\( e.g. \quad -1 \rightarrow 0, \; 1 \rightarrow 2, \; (-1,1) \rightarrow (-1+1, \; 2+2), \; (0, \; 2) \) \( (A1)(A1) \)

\( P \) is \( (3, \; 0) \) \( A1A1 \) \( N4 \)

\( \text{METHOD 2} \)

\( h(x) = 2(x-4)^2 - 2 \) \( (A1)(A1) \)

\( P \) is \( (3, \; 0) \) \( A1A1 \) \( N4 \)

[6 marks]

QUESTION 6

(a) (i) interchanging \( x \) and \( y \) (seen anywhere) \( M1 \)
\( e.g. \quad x = e^y \)

correct manipulation \( A1 \)
\( e.g. \quad \ln x = y + 3, \quad \ln y = x + 3 \)

\( f^{-1}(x) = \ln x - 3 \) \( AG \) \( N0 \)

(ii) \( x > 0 \) \( A1 \) \( N1 \)

(b) collecting like terms; using laws of logs \( (A1)(A1) \)
\( e.g. \quad \ln x - \ln \left( \frac{1}{x} \right) = 3, \quad \ln x + \ln x = 3, \quad \ln \left( \frac{1}{x} \right) = 3, \quad \ln x^2 = 3 \)

simplify \( (A1) \)
\( e.g. \quad \ln x = \frac{3}{2}, \quad x^2 = e^\pi \)

\( x = e^\pi \left\lfloor \sqrt{2} \right\rfloor \) \( A1 \) \( N2 \)

[7 marks]

QUESTION 7

attempt to substitute into formula \( V = \int \pi y^2 \, dx \) \( (M1) \)

integral expression \( A1 \)
\( e.g. \quad \frac{1}{2} \left[ \sqrt{2} \right]^2 \, x \)

correct integration \( (A1) \)
\( e.g. \quad \frac{1}{2} \, x \)

correct substitution \( V = \pi \left[ \frac{1}{2} \, x \right]^2 \) \( (A1) \)
equating their expression to \( 32 \pi \) \( M1 \)
\( e.g. \quad \pi \left[ \frac{1}{4} \, x \right]^2 = 32 \pi \)

\( a^2 = 64 \)
\( a = 8 \) \( A2 \) \( N2 \)

[7 marks]
SECTION B

QUESTION 8

(a) (i) \( x = 3 \cos \theta \)  \\
(ii) \( y = 3 \sin \theta \)  \\
\( [2 \text{ marks}] \)

(b) finding area  \\
\( e.g. \ A = 2x \times 2y, \ A = 8 \times \frac{1}{2}bh \)  \\
substituting  \\
\( e.g. \ A = 4 \times 3 \sin \theta \times 3 \cos \theta , \ 8 \times \frac{1}{2} \times 3 \cos \theta \times 3 \sin \theta \)  \\
\( A = 18(2 \sin \theta \cdot \cos \theta) \)  \\
\( A = 18 \sin 2\theta \)  \\
\( [3 \text{ marks}] \)

(c) (i) \( \frac{dA}{d\theta} = 36 \cos 2\theta \)  \\
(ii) for setting derivative equal to 0  \\
\( e.g. \ 36 \cos 2\theta = 0, \ \frac{dA}{d\theta} = 0 \)  \\
\( 2\theta = \frac{\pi}{2} \)  \\
\( \theta = \frac{\pi}{4} \)  \\
\( [3 \text{ marks}] \)

Total \( [13 \text{ marks}] \)

QUESTION 9

(a) (i) evidence of approach  \\
\( e.g. \ \vec{PO} + \vec{CQ} = \vec{OP} \)  \\
\( \vec{PO} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \)  \\
\( [1 \text{ mark}] \)

(ii) \( \vec{PR} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \)  \\
\( [1 \text{ mark}] \)

(b) METHOD 1  \\
choosing correct vectors \( \vec{PO} \) and \( \vec{PR} \)  \\
\( \frac{\vec{PO} \cdot \vec{PR}}{\|\vec{PO}\| \cdot \|\vec{PR}\|} \)  \\
\( \vec{PO} = \sqrt{2^2 + 1^2 + 1^2}, \ \vec{PR} = \sqrt{2^2 + 2^2 + 4^2} \)  \\
substituting into formula for angle between two vectors  \\
\( e.g. \ \cos \angle \vec{RPQ} = \frac{\sqrt{6} \cdot \sqrt{24}}{\sqrt{6} \cdot \sqrt{24}} \)  \\
simplifying to expression clearly leading to  \( \frac{1}{2} \)  \\
\( [3 \text{ marks}] \)

METHOD 2  \\
evidence of choosing cosine rule (seen anywhere)  \\
\( \frac{\vec{QR} \cdot \vec{PR}}{\|\vec{QR}\| \cdot \|\vec{PR}\|} \)  \\
\( \vec{QR} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \)  \\
\( \frac{\sqrt{5} \cdot \sqrt{6} + \sqrt{5} \cdot \sqrt{24} + \sqrt{5} \cdot \sqrt{12}}{\sqrt{5} \cdot \sqrt{6}} \)  \\
\( \cos \angle \vec{RPQ} = \frac{\sqrt{6} \cdot \sqrt{24}}{\sqrt{6} \cdot \sqrt{24}} \)  \\
\( \cos \angle \vec{RPQ} = \frac{\sqrt{6} \cdot \sqrt{24}}{\sqrt{6} \cdot \sqrt{24}} \)  \\
\( \cos \angle \vec{RPQ} = \frac{1}{2} \)  \\
\( [7 \text{ marks}] \)

continued …
Question 9 continued

(c) (i) METHOD 1

evidence of appropriate approach

\( \text{e.g. using } \sin^2 \text{RPQ} + \cos^2 \text{RPQ} = 1, \text{ diagram } \)

substituting correctly

\( \text{e.g. } \sin \text{RPQ} = \sqrt{1 - \left( \frac{1}{2} \right)^2} \)

\( \sin \text{RPQ} = \frac{\sqrt{3}}{\sqrt{2}} \left( -\frac{\sqrt{3}}{2} \right) \)

METHOD 2

since \( \cos \hat{P} = \frac{1}{2} \), \( \hat{P} = 60 \)

evidence of approach

\( \text{e.g. drawing a right triangle, finding the missing side } \)

\( \sin \hat{P} = \frac{\sqrt{3}}{2} \)

\( A1 \quad N3 \)

(ii) evidence of appropriate approach

\( \text{e.g. attempt to substitute into } \frac{1}{2} \sin \text{C} \)

correct substitution

\( \text{e.g. area } = \frac{\sqrt{3}}{2} \times \sqrt{3} + \frac{\sqrt{3}}{2} \)

\( \text{area } = 3\sqrt{3} \)

\( A1 \quad N2 \)

[6 marks]

Total [16 marks]

---

QUESTION 10

(a) METHOD 1

evidence of substituting \(-x\) for \(x\)

\( f(-x) = \frac{\frac{ax}{x^2 + 1}}{(-x)} = \frac{ax}{x^2 + 1} \quad AI \)

\( f(-x) = \frac{ax}{x^2 + 1} \quad AG \quad N0 \)

METHOD 2

\( y = -f(x) \) is reflection of \( y = f(x) \) in \( x \)-axis

and \( y = f(-x) \) is reflection of \( y = f(x) \) in \( y \)-axis

sketch showing these are the same

\( f(-x) = \frac{ax}{x^2 + 1} \quad AG \quad N0 \)

[2 marks]

(b) evidence of appropriate approach

\( \text{e.g. } f'(x) = 0 \)

to set the numerator equal to 0

\( \text{e.g. } 2ax(x^2 - 3) = 0 \) \( x^2 - 3 = 0 \)

\( (0, 0), \left( \pm \sqrt{3}, \pm \frac{3\sqrt{3}}{2} \right) \) (accept \( x = 0, y = 0 \) etc.)

\( A1AIAIAIAIAIAI \)

[7 marks]

continued ...
Question 10 continued

(c) (i) correct expression
   e.g. \( \frac{a}{3} \ln (x') + b \), \( \frac{a}{3} \ln 50 - \frac{a}{3} \ln 10 \), \( \frac{a}{3} (\ln 50 - \ln 10) \)
   area \( \frac{a}{2} \ln 5 \)  

   A1 A1 N2

(ii) METHOD 1
   recognizing the shift that does not change the area  
   e.g. \( \int_0^1 f(x)\,dx = \int_0^1 f(x)\,dx \) 

   recognizing that the factor of 2 doubles the area  
   e.g. \( \int_0^2 f(x)\,dx = 2 \int_0^1 f(x)\,dx = 2 \int_0^1 f(x)\,dx \) 
   \( \int_0^2 f(x)\,dx = a \ln 5 \) (i.e. 2 \times their answer to (c)(i))  

   A1 N3

METHOD 2
   changing variable
   let \( u = x - 1 \), so \( \frac{du}{dx} = 1 \)
   \( 2 \int f(u)\,du = \frac{2a}{2} \ln (u' + 1) + c \)  

   A1 N3

   [7 marks]

   Total [16 marks]
QUESTION 4

(a) 5

(b) METHOD 1

\[
\log_b \left( \frac{32^x}{8^y} \right) = \log_b 32^x - \log_b 8^y \\
= x \log_b 32 - y \log_b 8 \\
p = 5, q = -3 \quad \text{(accept } 5x - 3y) 
\]

METHOD 2

\[
\frac{32^x}{8^y} = \left( \frac{2^5}{2^3} \right)^x \\
= 2^{5x-3y} \\
\log (2^{5x-3y}) = 5x - 3y \\
p = 5, q = -3 \quad \text{(accept } 5x - 3y) 
\]

\[5 \text{ marks}\]

QUESTION 5

(a) METHOD 1

\[M = (M^T)^{-1} \quad (M1)\]

\[
M = \begin{pmatrix}
2 & 0 \\
0 & 1 \\
\end{pmatrix} \\
\text{AIAI N3} 
\]

METHOD 2

\[
\begin{pmatrix}
a & b \\
c & d \\
\end{pmatrix} = \begin{pmatrix} 5 & 0 \\
1 & 2 \\
\end{pmatrix} \\
\text{AI} 
\]

5a + b = 1, 2b + 0 = 1, 5c + d = 0, 2d = 1

\[
M = \begin{pmatrix}
0.2 & 0 \\
-0.1 & 0.5 \\
\end{pmatrix} \\
\text{AI N3} 
\]

(b) METHOD 1

evidence of appropriate approach \( \text{(M1)} \)

e.g. \( X = M^T B \)

\[
\begin{pmatrix}
s \\
y \\
\end{pmatrix} = \begin{pmatrix} 5 & 0 & 1 \\
1 & 2 & 1 \\
\end{pmatrix} \\
\text{AI} 
\]

\[
\begin{pmatrix}
s \\
y \\
\end{pmatrix} = \begin{pmatrix} 5 \\
15 \\
\end{pmatrix} \\
\text{AI N2} 
\]

METHOD 2

evidence of appropriate approach \( \text{(M1)} \)

e.g. \( \begin{pmatrix} 0.2 & 0 \\
-0.1 & 0.5 \\
\end{pmatrix} \begin{pmatrix} s \\
y \\
\end{pmatrix} = 1 \)

0.2x + 1 = 0, -0.1x + 0.5y = 7

\[
\begin{pmatrix}
s \\
y \\
\end{pmatrix} = \begin{pmatrix} 5 \\
15 \\
\end{pmatrix} \\
\text{AI N2} 
\]

\[6 \text{ marks}\]
QUESTION 6

(a) METHOD 1
\[ f''(x) = 3(x-3)^2 \]
A2 N2

METHOD 2
attempt to expand \((x-3)^3\)
\(e.g. f'(x) = x^3 - 9x^2 + 27x - 27\)
\(f'(3) = 3x^2 - 18x + 27\)
A1 N2

(b) \(f'(3) = 0, f''(3) = 0\)
A1 N1

(c) METHOD 1
\(f''\) does not change sign at \(P\)
evidence for this
R1 R1 N0

METHOD 2
\(f''\) changes sign at \(P\) so \(P\) is a maximum/minimum (i.e. not inflexion)
evidence for this
R1 R1 N0

METHOD 3
finding \(f(x) = \frac{1}{4}(x-3)^4 + c\) and sketching this function
indicating minimum at \(x = 3\)
R1 N0 [8 marks]

QUESTION 7

e^\int \left( \sqrt{3} \sin x + \cos x \right) dx = 0 \quad (A1)
e^\int = 0 \ not \ possible \ (seen \ anywhere) \quad (A1)
simplifying
\(e.g. \sqrt{3} \sin x + \cos x = 0, \sqrt{3} \sin x = -\cos x, \ \frac{\sin x}{-\cos x} = \frac{1}{\sqrt{3}}\) A1

\[ \tan x = -\frac{1}{\sqrt{3}} \quad A1 \]
\[ x = \frac{\pi}{6} \quad A2 N4 \]

OR

sketch of \(30^\circ, 60^\circ, 90^\circ\) triangle with sides 1, 2, \(\sqrt{3}\) A1
work leading to \(x = \frac{\pi}{6}\) A1
verifying \(\frac{\pi}{6}\) satisfies equation A1 N4 [6 marks]
QUESTION 8
(a) (i) \(-3e^{3x}\)  
\[A1 \quad N1\]
(ii) \(\cos\left(x - \frac{\pi}{3}\right)\)  
\[A1 \quad N1\]
(b) evidence of choosing product rule  
\(\text{e.g. } u'v + uv'\)  
\(\text{(M1)}\)
\[(A1)\]
\(\text{correct expression}\)  
\(\text{e.g. } -3e^{3x}\sin\left(x - \frac{\pi}{3}\right) + e^{3x}\cos\left(x - \frac{\pi}{3}\right)\)  
\[A1\]
\(\text{complete correct substitution of } x = \frac{\pi}{3}\)  
\(\text{(A1)}\)
\(\text{e.g. } -3e^{\frac{\pi}{3}}\sin\left(\frac{\pi}{3} - \frac{\pi}{3}\right) + e^{\frac{\pi}{3}}\cos\left(\frac{\pi}{3} - \frac{\pi}{3}\right)\)  
\[A1 \quad N3\]  
\[\text{[6 marks]}\]

SECTION B
QUESTION 9
(a) \[
\begin{array}{ccc}
3,9 & 4,9 & 5,9 \\
10,3 & 10,4 & 10,5 \\
3,10 & 4,10 & 5,10 \\
\end{array}
\]
\[A2 \quad N2\]  
\[\text{[2 marks]}\]
(b) 12, 13, 14, 15 (accept 12, 13, 13, 14, 14, 15, 15)  
\[A2 \quad N2\]  
\[\text{[2 marks]}\]
(c) \(P(12) = \frac{1}{9}, P(13) = \frac{3}{9}, P(14) = \frac{3}{9}, P(15) = \frac{2}{9}\)  
\[A2 \quad N2\]  
\[\text{[2 marks]}\]
(d) correct substitution into formula for \(E(X)\)  
\(\text{e.g. } E(S) = 12 \times \frac{1}{9} + 13 \times \frac{3}{9} + 14 \times \frac{3}{9} + 15 \times \frac{2}{9}\)  
\[A2 \quad N2\]  
\[\text{[3 marks]}\]
(e) METHOD 1  
correct expression for expected gain \(E(A)\) for 1 game  
\(\text{e.g. } \frac{4}{9} \times 50 + \frac{5}{9} \times 30\)  
\[A1\]
\(E(A) = \frac{50}{9}\)  
\(\text{amount at end} = \text{expected gain for } 1 \text{ game} \times 36\)  
\(= 200 \text{ (dollars)}\)  
\[A1 \quad N2\]
METHOD 2  
attempt to find expected number of wins and losses  
\(\text{e.g. } \frac{4}{9} \times 36, \frac{5}{9} \times 36\)  
\[\text{(M1)}\]
\(\text{attempt to find expected gain } E(G)\)  
\(\text{e.g. } 16 \times 50 - 30 \times 20\)  
\(E(G) = 200 \text{ (dollars)}\)  
\[A1 \quad N2\]  
\[\text{[3 marks]}\]

Total [12 marks]
QUESTION 10

(a) \( L_1: \mathbf{r} = \begin{bmatrix} 8 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \)

[2 marks]

(b) evidence of equating \( \mathbf{r} \) and \( \mathbf{OA} \)

\( e.g. \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix} \) \( \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \), \( A = r \)

one correct equation

\( e.g. 6 = 2 + 2t, 2 = 4 - t, 9 = -1 + 5t \)

so \( A \) lies on \( L_1 \)

[4 marks]

(c) (i) evidence of approach

\( e.g. \begin{bmatrix} 2 \\ 2 \\ 8 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \) \( L_1 = L_2 \)

one correct equation

\( e.g. 2 + 2s = 8, 2 - s = 1, -1 + 5s = t \)

attempt to solve

finding \( s = 3 \)

[7 marks]

(ii) evidence of appropriate approach

\( e.g. \mathbf{AB} = \mathbf{AD} + \mathbf{OB}, \mathbf{AB} = \mathbf{OB} - \mathbf{OA} \)

\( \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \)

[3 marks]

(d) evidence of appropriate approach

\( e.g. \mathbf{AB} = \mathbf{DC} \)

correct values

\( e.g. \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} + s \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix} \)

\( \begin{bmatrix} 0 \\ 2 \\ -9 \end{bmatrix} \)

[3 marks]

Total [16 marks]
QUESTION 11

Note: In this question, do not penalize absence of units.

(a) (i) \[ s = \int (40 - at) \, dt \] 
\[ s = 40t - \frac{1}{2}at^2 + c \] 
substituting \( s = 100 \) when \( t = 0 \) \( (c = -100) \) 
\[ s = 40t - \frac{1}{2}at^2 + 100 \] 
\[ A1 \] 
\[ A1 \] 

(ii) \[ s = 40t - \frac{1}{2}at^2 \] 
\[ A1 \] 
\[ N1 \] 

[6 marks]

(b) (i) stops at station, so \( v = 0 \) 
\[ t = \frac{40}{a} \] (seconds) 
\[ A1 \] 
\[ N2 \] 

(ii) evidence of choosing formula for \( s \) from (a) (ii) 
substituting \( t = \frac{40}{a} \) 
\[ e.g. \ 40 \cdot \frac{40}{a} - \frac{1}{2}a \cdot \frac{40^2}{a^2} \] 
setting up equation 
\[ e.g. \ 500 = s, \ 500 = 40 \cdot \frac{40}{a} - \frac{1}{2}a \cdot \frac{40^2}{a^2}, \ 500 = 1600 - \frac{800}{a} a \] 
evidence of simplification to an expression which obviously leads to \( a = \frac{8}{5} \) 
\[ A1 \] 
\[ AG \] 
\[ N0 \] 

[6 marks]

continued ...

Question 11 continued

(c) METHOD 1
\[ v = 40 - 4t, \ \text{stops when} \ v = 0 \] 
\[ 40 - 4t = 0 \] 
\[ t = 10 \] 
\[ A1 \] 
substituting into expression for \( s \) 
\[ s = 40 \cdot 10 - \frac{1}{2} \cdot 4 \times 10^2 \] 
\[ s = 200 \] 
since \( 200 < 500 \) (allow FT on their \( s \), if \( s < 500 \)) 
\[ R1 \] 
train stops before the station 
\[ AG \] 
\[ N0 \] 

METHOD 2
from (b) \( t = \frac{40}{a} = 10 \) 
substituting into expression for \( s \) 
\[ e.g. \ s = 40 \cdot \frac{40}{a} - \frac{1}{2} \cdot 4 \times 10^2 \] 
\[ s = 200 \] 
\[ A1 \] 
\[ R1 \] 
train stops before the station 
\[ AG \] 
\[ N0 \] 

METHOD 3
\( a \) is deceleration 
\[ 4 \times \frac{8}{5} \] 
\[ A1 \] 
so stops in shorter time 
so less distance travelled 
so stops before station 
\[ AG \] 
\[ N0 \] 

[5 marks]

Total [17 marks]
SECTION A

QUESTION 1

(a) evidence of setting function to zero
   \( e.g. f(x) = 0, 8x = 2x^2 \)   \( (M1) \)
   evidence of correct working
   \( e.g. 0 = 2x(4-x), -\frac{8 \pm \sqrt{64}}{4} \)   \( A1 \)
   \( x \)-intercepts are at 4 and 0 (accept (4, 0) and (0, 0), or \( x = 4, x = 0 \)   \( A1A1 \) \( N1N1 \)

(b) (i) \( x = 2 \) (must be equation)   \( A1 \) \( N1 \)
    (ii) substituting \( x = 2 \) into \( f(x) \)

   \( y = 8 \)   \( (M1) \)   \( A1 \) \( N2 \)

[7 marks]

QUESTION 2

(a) \( WP = \begin{pmatrix} 13 \\ 5 \\ 6 \end{pmatrix} \)   \( A1A1A1 \) \( N3 \)

Note: Award A1 for each correct element.

(b) Note: The first two steps may be done in any order.

   subtracting
   \( \begin{pmatrix} 26 \\ 10 \end{pmatrix} \) \( 2WP \)
   \( e.g. \begin{pmatrix} 26 \\ 10 \end{pmatrix} \)

   multiplying \( WP \) by 2
   \( e.g. \begin{pmatrix} 26 \\ 10 \end{pmatrix} \)
   \( S = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} \)   \( A1 \) \( N2 \)

[6 marks]

QUESTION 3

(a) evidence of expanding
   \( e.g. 2x^4 + 4(2x^2) + 6(4x^2 + 4(2)x^2) + 4(4 + 4x + x^2)(4 + 4x + x^2) \)
   \( (2 + x)^2 = 16 + 32x + 24x^2 + 8x^3 + x^4 \)   \( A2 \) \( N2 \)

(b) finding coefficients 24 and 1
    \( (A1)(A1) \)
    term is \( 25x^5 \)   \( A1 \) \( N3 \)

[6 marks]

QUESTION 4

(a) \( \tan \theta = \frac{3}{4} \) (do not accept \( \frac{1}{4} \))   \( A1 \) \( N1 \)

(b) (i) \( \sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5} \)
    \( (A1)(A1) \)
    correct substitution
    \( e.g. \sin 2\theta = \left( \frac{3}{5} \right) \left( \frac{4}{5} \right) \)   \( A1 \)
    \( \sin 2\theta = \frac{24}{25} \)   \( A1 \) \( N3 \)

(ii) correct substitution
    \( e.g. \cos 2\theta = 1 - 2\sin^2 \theta \)
    \( = \left( \frac{1}{5} \right) \left( \frac{4}{5} \right) - 2 \left( \frac{3}{5} \right)^2 \)
    \( \cos 2\theta = \frac{7}{25} \)   \( A1 \) \( N1 \)

[7 marks]
QUESTION 5

(a) (i) \( p = 0.2 \) \( \text{AI} \) \( \text{NI} \)
(ii) \( q = 0.4 \) \( \text{AI} \) \( \text{NI} \)
(iii) \( r = 0.1 \) \( \text{AI} \) \( \text{NI} \)

(b) \( P(A|B) = \frac{2}{3} \) \( \text{A2} \) \( \text{N2} \)

Note: Award \( \text{AI} \) for an unfinished answer such as \( 0.2 \) or \( 0.3 \).

(c) valid reason \( \text{R1} \)
\[
\frac{2}{3} \neq 0.5, 0.35 \neq 0.3
\]
thus, \( A \) and \( B \) are not independent \( \text{AG} \) \( \text{N0} \) [6 marks]

QUESTION 6

attempt to set up integral expression \( \text{M1} \)
\[
e.g. \int \sqrt{6-4x^2} \, dx, \int [0,16-4x^2] \, dx
\]
\[
\int 16 \, dx = 16x, \frac{4}{3}x^3 \text{ seen anywhere}
\]
evidence of substituting limits into the integrand \( \text{M1} \)
\[
e.g. \left[ 32 \frac{32}{3} \right] - \left[ -32 + \frac{32}{3} \right] \quad 64 \quad \frac{64}{3}
\]
volume \( \frac{128\pi}{3} \) \( \text{A2} \) \( \text{N3} \) [6 marks]

QUESTION 7

(a) interchanging \( x \) and \( y \) (seen anywhere) \( \text{(M1)} \)
e.g. \( x = \log_2 y \) (accept any base)

evidence of correct manipulation \( \text{AI} \)
\[
e.g. \ 3^x = \sqrt{y}, \ 3^y = 2^x, \ x = \frac{1}{2} \log_2 y, \ 2y = \log_2 x
\]
\[
f^{-1}(x) = 3^x
\]
\[
y > 0, \ f^{-1}(x) > 0
\]

(b) \( y > 0, \ f^{-1}(x) > 0 \) \( \text{AI} \) \( \text{NI} \)

(c) METHOD 1

finding \( g(2) = \log_2 2 \) (seen anywhere) \( \text{AI} \)

try to substitute \( \text{(M1)} \)
e.g. \( (f^{-1} + g)(2) = 3^{\log_2 2} \)
evidence of using log or index rule \( \text{(A1)} \)
\[
e.g. \ (f^{-1} + g)(2) = 3^{\log_2 2}, \ 3^{\frac{1}{2}}
\]
\[
(f^{-1} + g)(2) = 4
\]

METHOD 2

attempt to form composite (in any order) \( \text{(M1)} \)
e.g. \( (f^{-1} + g)(1) = 3^{\frac{1}{2}} \)
evidence of using log or index rule \( \text{(A1)} \)
\[
e.g. \ (f^{-1} + g)(1) = 3^{\log_2 2}, \ 3^{\frac{1}{2}}
\]
\[
(f^{-1} + g)(2) = 4
\]

\[\text{[7 marks]}\]
SECTION B

QUESTION 8

(a) \( f(x) = x^2 - 2x - 3 \)

- evidence of solving \( f'(x) = 0 \)
  - \( x^2 - 2x - 3 = 0 \)
- evidence of correct working
  - \( x + 1)(x - 3) = 0 \)
- \( x = -1 \) (ignore \( x = 3 \))
- evidence of substituting their negative value into \( f(x) \)
  - \( y = \frac{2}{3} \)
- coordinates are \( (-1, 5), \left(\frac{1}{3}, -1\right) \) \( \[8 \text{ marks}\] \)

(b) (i) \((-3, -9)\)
  - \( (1, -4) \)
  - reflection gives \((3, 9)\)
  - stretch gives \( \left(\frac{3}{2}, 9\right) \) \( \[6 \text{ marks}\] \)

Total \[14 \text{ marks}\]

QUESTION 9

(a) \( \frac{d}{dx} \sin x = \cos x, \quad \frac{d}{dx} \cos x = -\sin x \) (seen anywhere)

- evidence of using the quotient rule
  - \( \frac{\sin x \cdot \cos x \cdot (\cos x) - \sin x \cdot \sin x \cdot (\cos x)}{\sin^2 x} \)
  - \( f'(x) = \frac{-1}{\sin^2 x} \)
  - \( f''(x) = \frac{2\cos x}{\sin^3 x} \)

- appropriate approach
  - \( \frac{d}{dx} (\sin^2 x) = 2\sin x \cdot \cos x \)
  - \( f''(x) = 2\sin x \cdot \cos x \) \( \[5 \text{ marks}\] \)

Note: Award \( A1 \) for \( 2\sin^2 x, A1 \) for \( \cos x \).

(b) METHOD 1

- derivative of \( \sin^2 x = 2\sin x \cdot \cos x \) (seen anywhere)
  - \( f''(x) = \frac{2\cos x \cdot \cos x}{\sin^3 x} \)
  - \( f''(x) = 2\cos x \cdot \cos x \)
  - \( f''(x) = \frac{2\cos x}{\sin^3 x} \)
  - \( f''(x) = \frac{2\cos x}{\sin^3 x} \)
  - \( f''(x) = (\sin^2 x) \)

- \( A1 \) \( A1 \) \( N3 \)

- total \[13 \text{ marks}\]

(c) evidence of substituting \( \pm \frac{\pi}{2} \)

- \( p = -1, q = \frac{2\cos x}{\sin^3 x} \)

- second derivative is zero, second derivative changes sign

- total \[13 \text{ marks}\]
QUESTION 10

(a) any correct equation in the form \( r = a + tb \) (accept any parameter) \( A2 \ N2 \)

\[
\begin{pmatrix}
-5 \\
2 \\
25
\end{pmatrix} + t
\begin{pmatrix}
2 \\
1 \\
-8
\end{pmatrix}
\]

Note: Award \( A1 \) for \( a + tb \), \( A1 \) for \( L = a + tb \), \( A0 \) for \( r = b + u \) \[2 \text{ marks}\]

(b) recognizing scalar product must be zero (seen anywhere) \( R1 \)

\( e.g. \ a \cdot b = 0 \)

evidence of choosing direction vectors \( \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}, \begin{pmatrix} -7 \\ 2 \\ k \end{pmatrix} \) \( \text{(A1)/(A1)} \)

correct calculation of scalar product \( \text{(A1)} \)

\[
2(-7) + 1(-2) - 8k
\]

simplification that clearly leads to solution \( \text{(A1)} \)

\( e.g. \ -16 - 8k = 0 \)

\( k = -2 \) \( AG \ N0 \) \[5 \text{ marks}\]

(c) evidence of equating vectors \( \text{(M1)} \)

\[
\begin{pmatrix}
-3 \\
-1 \\
28
\end{pmatrix} = \begin{pmatrix}
2 \\
1 \\
3
\end{pmatrix} + t \begin{pmatrix}
2 \\
1 \\
5
\end{pmatrix}
\]

any two correct equations \( A1 \) \( \text{A1} \)

\( e.g. \ -3 + 2p = 5 - 7q, \ -1 + p = -2q, \ 25 - 8p = 3 - 2q \)

attempting to solve equations \( \text{(M1)} \)

finding one correct parameter \( p = -3, q = 2 \) \( A1 \)

the coordinates of \( A \) are \((-9, -4, -1)\) \( A1 \ N3 \) \[6 \text{ marks}\]

continued ...

Question 10 continued

(d) (i) evidence of appropriate approach \( \text{(M1)} \)

\[
\begin{pmatrix} 1 \\ -1 \\ 26 \end{pmatrix}
\]

\( \overrightarrow{AB} = \begin{pmatrix} 8 \\ -4 \\ -1 \end{pmatrix} \)

\( A1 \ N2 \)

(ii) finding \( \overrightarrow{AC} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \) \( A1 \)

evidence of finding magnitude \( \text{(M1)} \)

\( e.g. \ |\overrightarrow{AC}| = \sqrt{2^2 + 2^2} \)

\( |\overrightarrow{AC}| = \sqrt{57} \) \( A1 \ N3 \) \[5 \text{ marks}\]

Total \[18 \text{ marks}\]
SECTION A

QUESTION 1
(a) evidence of setting function to zero
\( e.g. \, f(x) = 0, \, 8x = 2x^3 \) (M1)
evidence of correct working
\( e.g. \, 0 = 2x(4 - x), \, \frac{8 \pm \sqrt{64}}{-4} \) (A1)
x-intercepts are at 4 and 0 (accept (4, 0) and (0, 0), or \( x = 4, \, x = 0 \)) (AI) (N1)
(b) (i) \( x \neq 2 \) (must be equation) (AI) (N1)
(ii) substituting \( x = 2 \) into \( f(x) \) (M1)
y = 8 (AI) (N1)

QUESTION 2
(a) \( WP = \begin{pmatrix} 13 \\ 5 \\ 6 \end{pmatrix} \) (A1) (AI) (N3)
(b) \( \text{Note: Award AI for each correct element.} \)

Note: The first two steps may be done in any order.
subtracting
\( \begin{pmatrix} 12 \\ 10 \end{pmatrix} \) \( = 2WP \) (AI)
multiplying \( WP \) by 2
\( \begin{pmatrix} 26 \\ 10 \end{pmatrix} \) (AI)
\( S = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} \) (AI) (N2)

[6 marks]

QUESTION 3
(a) evidence of expanding
\( e.g. \, 2^x + 4(2^x) x + 6(2^x) x^2 + 4(2) x^3 + x^4, \, (4x + 4x^3)(4x + x^3) \) (M1)
\( 2 + x^2 = 16 + 32x + 24x^2 + 8x^4 + x^6 \) (A2) (N2)
(b) finding coefficients 24 and 1 (AI)(AI)
term is \( 25x^2 \) (AI) (N3)
[6 marks]

QUESTION 4
(a) \( \tan \theta = \frac{3}{4} \) (do not accept \( \frac{4}{3} \)) (AI) (N1)
(b) (i) \( \sin \theta = \frac{3}{5}, \, \cos \theta = \frac{4}{5} \) (AI)(AI)
correct substitution (AI)
\( e.g. \, \sin 2\theta = \left( \frac{3}{5} \right) \left( \frac{4}{5} \right) \) (A1)
\( \sin 2\theta = \frac{24}{25} \) (AI) (N3)
(ii) correct substitution
\( e.g. \, \cos 2\theta = 1 - 2 \left( \frac{3}{5} \right)^2 = 1 - 2 \left( \frac{9}{25} \right) = \frac{7}{25} \) (AI) (N1)
[7 marks]
QUESTION 5

(a) (i) \( p = 0.2 \)
\[ A1 \]
(ii) \( q = 0.4 \)
\[ A1 \]
(iii) \( r = 0.1 \)
\[ A1 \]
(b) \( P(A|B) = \frac{2}{3} \)
\[ A2 \]

Note: Award \( A1 \) for an unfinished answer such as 0.2

(c) valid reason
\[ R1 \]
\[ e.g. \frac{2}{3} \neq 0.5, 0.35 \neq 0.3 \]
thus, \( A \) and \( B \) are not independent
\[ AG \]

QUESTION 6

attempt to set up integral expression
\[ M1 \]
e.g. \( \pi \int \sqrt{6-4x^2} \, dx, \ 2\pi \int (6-4x^2)^{\frac{1}{2}} \, dx \)
\[ \int 16 \, dx = 16x, \int 4x^3 \, dx = \frac{4x^4}{3} \quad (\text{seen anywhere}) \]
evidence of substituting limits into the integrand
\[ M1 \]
e.g. \( \left[ 32 \frac{32}{3} \right] \left[ -32 + \frac{32}{3} \right], 64 \quad \frac{64}{3} \]
volume \( \frac{128\pi}{3} \)
\[ A2 \]

[6 marks]

QUESTION 7

(a) interchanging \( x \) and \( y \) (seen anywhere)
\[ (M1) \]
e.g. \( x = \log \sqrt{y} \) (accept any base)
evidence of correct manipulation
\[ A1 \]
e.g. \( 3' = \sqrt{y}, 3' = \sqrt{y}, x = \log_2(y), 2y = \log_2x \)
\( f'(x) = 3'' \)
\[ AG \]

(b) \( y > 0, f'(x) > 0 \)
\[ A1 \]

(c) METHOD 1

finding \( g(2) = \log_2 2 \) (seen anywhere)
\[ A1 \]
attempt to substitute
\[ (M1) \]
e.g. \( (f^{-1} + g)(2) = 3^{\log_2 2} \)
evidence of using log or index rule
\[ (A1) \]
e.g. \( (f^{-1} + g)(2) = 3^{\log_2 2}, 3^{\log_2 2} \)
\( f^{-1}(2) = 4 \)
\[ A1 \]

METHOD 2

attempt to form composite (in any order)
\[ (M1) \]
e.g. \( (f^{-1} + g)(1) = 3^{\log_2 2} \)
evidence of using log or index rule
\[ (A1) \]
e.g. \( (f^{-1} + g)(1) = 3^{\log_2 2}, 3^{\log_2 2} \)
\( f^{-1}(2) = 4 \)
\[ A1 \]

[7 marks]
SECTION B

QUESTION 8

(a) \( f(x) = x^3 - 2x - 3 \)\( A1 \)\( A1 \)
evidence of solving \( f'(x) = 0 \)
e.g. \( x^2 - 2x - 3 = 0 \)
evidence of correct working\( A1 \)
e.g. \( (x+1)(x-3), \frac{2\pm \sqrt{14}}{2} \)
x = -1 (ignore \( x = 3 \))\( A1 \)
evidence of substituting their negative \( x \)-value into \( f(x) \)
e.g. \( \frac{1}{3}(-1)^3 - (-1)^2 - 3(-1) = -1 + 3 \)
y = \( \frac{2}{3} \)\( N3 \)
coordinates are \( \left( -1, \frac{2}{3} \right) \)\( [8 \text{ marks}] \)

(b) (i) \((-3, -9)\)\( A1 \)\( N1 \)
(ii) \((1, -4)\)\( A1 \)\( N2 \)
(iii) reflection gives \((3, 9)\)\( A1 \)\( N3 \)
stretch gives \( \left( \frac{3}{2}, 9 \right) \)\( [6 \text{ marks}] \)
Total \([14 \text{ marks}] \)

QUESTION 9

(a) \( \frac{d}{dx} \sin x = \cos x, \frac{d}{dx} \cos x = -\sin x \) (seen anywhere)\( (A1)/(A1) \)
evidence of using the quotient rule\( M1 \)
correct substitution\( A1 \)
e.g. \( \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x} = -\frac{\sin^2 x - \cos^2 x}{\sin^2 x} \)
\( f'(x) = \frac{-\sin^2 x}{\sin^2 x} \)
\( f'(x) = -1 \)
\( \sin^2 x \)\( A1 \)\( A1 \)\( N3 \)
[5 marks]

(b) METHOD 1

appropriate approach\( (M1) \)
e.g. \( f'(x) = -(\sin x)^2 \)
\( f''(x) = 2(\sin^2 x)(\cos x) \left( \frac{2\cos x}{\sin^3 x} \right) \)
\( A1 \)\( A1 \)\( N3 \)
Note: Award \( A1 \) for \( 2\sin^2 x, A1 \) for \( \cos x \).
[3 marks]

METHOD 2
derivative of \( \sin^2 x = 2\sin x \cos x \) (seen anywhere)\( A1 \)
evidence of choosing quotient rule\( M1 \)
e.g. \( u = -1, v = \sin^2 x, f''(x) = \frac{\sin^2 x(0 - (-1)2\sin x \cos x)}{(\sin^3 x)^2} \)
\( f''(x) = \frac{2\sin x \cos x}{(\sin^3 x)^2} \)
\( A1 \)\( N3 \)
[3 marks]

(c) evidence of substituting \( \frac{\pi}{2} \)\( M1 \)
e.g. \( \frac{-1}{\sin \frac{\pi}{2}} = \frac{2\cos\frac{\pi}{2}}{\sin \frac{\pi}{2}} \)
\( p = -1, q = 0 \)\( A1 \)\( A1 \)\( N1 \)\( N1 \)
[3 marks]

(d) second derivative is zero, second derivative changes sign\( R1 \)\( R1 \)
[2 marks]
Total \([13 \text{ marks}] \)
QUESTION 10

(a) any correct equation in the form \( r = a + tb \) (accept any parameter) \[ A2 \] \[ N2 \]

\[ e.g. \quad r = \begin{pmatrix} -5 \\ 2 \\ 25 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix} \]

Note: Award \( A1 \) for \( a + tb \), \( A1 \) for \( L \parallel a + tb \), \( A0 \) for \( r = b + su \). \[ 2 \text{ marks} \]

(b) recognizing scalar product must be zero (seen anywhere) \[ R1 \]

\[ e.g. \quad a \cdot b = 0 \]

evidence of choosing direction vectors \[ \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}, \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix} \] \( (A1)/(A1) \)

correct calculation of scalar product \[ e.g. \quad 2(-7) + 1(-2) -8k \] \( A1 \)

simplification that clearly leads to solution \( e.g. -16 -8k \), \( -16 -8k = 0 \)

\( k = -2 \) \[ A1 \]

\[ k \] \[ AG \]

\[ N0 \] \[ 5 \text{ marks} \]

(c) evidence of equating vectors \( \begin{pmatrix} -3 \\ -1 \\ -5 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ -7 \\ -2 \end{pmatrix} \] \( (M1) \)

any two correct equations \[ e.g. -3 + 2p = 5 -7q, -1 + p = -2q, -25 -8p = 3 -2q \]

attempts to solve equations \( (M1) \)

finding one correct parameter \( p = -3, q = 2 \) \( A1 \)

the coordinates of \( A \) are \((-9, -4, -3)\) \[ A1 \]

\[ N3 \] \[ 6 \text{ marks} \]

(d) (i) evidence of appropriate approach \( (M1) \)

\[ e.g. \quad \overrightarrow{OA} = \overrightarrow{AB} - \overrightarrow{OB} = \begin{pmatrix} -8 \\ 4 \\ 5 \end{pmatrix} \]

\[ \overrightarrow{AB} = \begin{pmatrix} 1 \\ -1 \\ 26 \end{pmatrix} \]

\[ A1 \]

\[ N2 \]

(ii) finding magnitude \( \overrightarrow{AC} \)

\[ e.g. \quad |\overrightarrow{AC}| = \sqrt{7^2 + 2^2 + 2^2} \] \( (M1) \)

\[ |\overrightarrow{AC}| = \sqrt{57} \] \[ A1 \]

\[ N3 \] \[ 5 \text{ marks} \]

Total \[ 18 \text{ marks} \]
SECTION A

QUESTION 1

(a) \(2, 4) \), \(qr = \) or \((4, 2) \), \(qr = \) \(A1 A1 N2\)

(b) \(1 \times \) (must be an equation) \(A1 N1\)

(c) substituting \((0, 4)\) into the equation (M1) e.g. \(4(0 - 2) = 0 - 4\), \(4(4) = 2\), \(pp = - - - - - - \)

correct working towards solution (A1) e.g. \(48 = 41\), \(82\) \(A1 N2\)

[6 marks]

QUESTION 2

(a) evidence of appropriate approach (M1) e.g. \(BC = BA + AC\), \(\begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} = -1 \) \(A1 N2\)

(b) attempt to find the length of \(AB\) (M1) \(|AB| = \sqrt{6^2 + (-2)^2 + 7^2} = \sqrt{36 + 4 + 49} = \sqrt{89} \) \(A1\)

unit vector is \(\frac{1}{\sqrt{79}} \begin{pmatrix} 6 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 6/\sqrt{79} \\ 2/\sqrt{79} \\ 7/\sqrt{79} \end{pmatrix} \) \(A1 N2\)

(c) recognizing that the dot product or \(\cos \theta\) being 0 implies perpendicular (M1) correct substitution in a scalar product formula \(A1\)

e.g. \((6)(-2) + (-2)(-3) + (3)(2)\), \(\cos \theta = \frac{12 + 6 + 6}{7\sqrt{79}}\)

correct calculation \(A1\)
e.g. \(\vec{A}\vec{B} \cdot \vec{A}\vec{C} = 0\), \(\cos \theta = 0\)

therefore, they are perpendicular \(AG N0\) [8 marks]

QUESTION 3

(a) evidence of multiplying \(e.g.\) one correct element \(AB = \begin{pmatrix} -15 \\ -1 \end{pmatrix} \) \(A1 A1 N3\)

(b) METHOD 1 evidence of multiplying by \(A\) (on left or right) (M1) \(e.g.\) \(A\vec{X} = AB\), \(X = AB\) \(A1 N2\)

\(\begin{pmatrix} -15 \\ 5 \end{pmatrix}\) (accept \(x = -15, y = 5\)) \(A1 N2\)

METHOD 2 attempt to set up a system of equations (M1) \(e.g.\) \(4x + 2y = -5, -3x + y = 5\)

\(\begin{pmatrix} -15 \\ 5 \end{pmatrix}\) (accept \(x = -15, y = 5\)) \(A1 N2\) [5 marks]

QUESTION 4

(a) \(\int \frac{\sqrt{2}}{2} = \cos \pi\) \((A1)\) \(-1\) \(A1 N2\)

(b) \(g(f(x)) = g(-1) + 2(-1)^2 - 1\) \((A1)\) \(-1\) \(A1 N2\)

(c) \((g \circ f)(x) = 2(2x) = 1\) \((A1)\) \((-2 \cos^2(2x) - 1)\) \(\cos \theta = \cos^2(2x)\) (seen anywhere) \((M1)\)

\((g \circ f)(x) = \cos 4x\) \(k = 4\) \(A1 N2\) [7 marks]
QUESTION 5

gradient of tangent = 8 (seen anywhere) \( f'(x) = 4kx^3 \) (seen anywhere) \( A1 \)

recognizing the gradient of the tangent is the derivative \( MA1 \)

setting the derivative equal to 8 \( A1 \)

\[ 3^k x^4 = 8, k x^2 = 2 \]

substituting \( x = 1 \) (seen anywhere) \( k = 2 \) \( MA1 \)

[6 marks]

QUESTION 6

recognizing \( \log a + \log b = \log ab \) (seen anywhere) \( MA1 \)

\( e.g. \log_a(x(x-2)), x^2 = 2x \) \( A1 \)

recognizing \( \log_b x = a \Rightarrow x^a = b \) (seen anywhere) \( A1 \)

\( e.g. \quad 2^x = 8 \)

correct simplification \( A1 \)

\( e.g. \quad x(x-2) = 2, x^2 = 2x = 8 \)

evidence of correct approach to solve \( MA1 \)

e.g. factorizing, quadratic formula \( A1 \)

correct working \( A1 \)

\( e.g. \quad (x-4)(x+2) = \frac{2+\sqrt{6}}{2}, x = 4 \) \( A2 \)

[7 marks]

QUESTION 7

(a) \( x \)-intercepts at \(-3, 0, 2 \) \( A2 \)

(b) \(-3 < x < 0, 2 < x < 3 \) \( A1A1 \)

(c) correct reasoning \( R2 \)

\( e.g. \) the graph of \( f \) is \textit{concave-down} (accept convex), the first derivative is decreasing therefore the second derivative is negative \( AG \)

[6 marks]

SECTION B

QUESTION 8

(a) \( \text{substituting into the second derivative} \quad MA1 \)

\( \text{\( e.g. \quad 3\left(\frac{4}{3}\right)^2 - 1 \)} \)

\( f'' \left(\frac{4}{3}\right) = -5 \quad A1 \)

since the second derivative is negative, \( B \) is a maximum \( R1 \) \( N0 \)

[3 marks]

(b) \( \text{setting} \quad f'(x) = 0 \quad (MA1) \)

\( \text{evidence of substituting} \ x = 2 \quad \text{or} \ x = -\frac{4}{3} \) \( (MA1) \)

\( e.g. \quad (2)^3 + \frac{4}{3} \quad \text{correct substitution} \quad A1 \)

\( e.g. \quad 32 \times 11 \times 22 \quad \text{correct simplification} \quad A1 \)

\( 64 + 2 = 0, \frac{4}{3} + \frac{4}{3} + p = 0, 4 + p + 0 = 0 \quad AG \) \( N0 \)

[4 marks]

(c) \( \text{evidence of integration} \quad (MA1) \)

\( f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^4 + c \quad A1A1A1 \)

\( \text{\( \text{\substituting} \quad (2, 4) \quad \text{or} \quad \left(\frac{4}{3}, \frac{35}{27}\right) \quad \text{into their expression} \)} \quad (MA1) \)

\( \text{correct equation} \quad A1 \)

\( e.g. \quad \frac{1}{2}x^3 - \frac{1}{2}x^4 + c = 4, \frac{1}{2}x^3 - \frac{1}{2}x^4 + c = 4, 4 - 2 \times 8 + c = 4 \)

\( f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^4 + 10 \quad A1 \)

[7 marks]

Total [14 marks]
QUESTION 9

(a) (i) \( \frac{7}{24} \)  
A1 N1

(ii) evidence of multiplying along the branches  
(M1)

e.g. \( \frac{2}{3}, \frac{5}{8}, \frac{1}{3}, \frac{7}{8} \)

adding probabilities of two mutually exclusive paths  
(M1)

e.g. \( \left( \frac{1}{4}, \frac{3}{8} \right) + \left( \frac{2}{3}, \frac{1}{2} \right) \)

\( P(F) = \frac{13}{24} \)  
A1 N2

(b) (i) \( \frac{1}{3} \times \frac{1}{8} \)  
A1

(ii) recognizing this is \( P(E|F) \)  
(M1)

e.g. \( \frac{1}{24}, \frac{168}{312} \)  
A2 N3

\[ \text{[5 marks]} \]

(c) \( X \) (cost in euros)

<table>
<thead>
<tr>
<th>Cost (euros)</th>
<th>Probability ( P(X) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \frac{1}{9} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{4}{9} )</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{4}{9} )</td>
</tr>
</tbody>
</table>

\[ \text{A2A4 N3 [3 marks]} \]

(d) correct substitution into \( E(X) \) formula  
(M1)

e.g. \( 0 \times \frac{1}{9} + 3 \times \frac{4}{9} + 6 \times \frac{4}{9} = \frac{24}{9} \)

\( E(X) = 4 \) (euros)  
A1 N2

\[ \text{[2 marks]} \]

Total [14 marks]

QUESTION 10

(a) (i) \( \sin x = 0 \)
\( x = 0, x = \pi \)  
A1 A1 A1 N2

(ii) \( \sin x = -1 \)
\( x = \frac{3\pi}{2} \)  
A1 N1

\[ \text{[5 marks]} \]

(b) \( \frac{3\pi}{2} \)  
A1 N1

\[ \text{[1 mark]} \]

(c) evidence of using anti-differentiation  
(M1)

e.g. \( \int (6 + 6\sin x) \)  
correct integral \( 6x - 6\cos x \) (seen anywhere)  
A1A1

correct substitution  
(M3)

e.g. \( 6 \times \left( \frac{3\pi}{2} \right) - 6\cos0, 9\pi - 6 \)
\( k = 9\pi + 6 \)  
A1A1 N3

\[ \text{[6 marks]} \]

(d) translation of \( \left( \frac{\pi}{2}, 0 \right) \)  
A1A1 N2

\[ \text{[2 marks]} \]

(e) recognizing that the area under \( g \) is the same as the shaded region in \( f \)  
(M3)

\[ p = \frac{2}{3}, p = 0 \]  
A1A1 N3

\[ \text{[3 marks]} \]

Total [17 marks]
SECTION A

QUESTION 1

(a) (i) \[ AB = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \] (4 marks) A2 N2

(ii) \[ A' = \frac{1}{4} \begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix} \] A1 N1

(b) METHOD 1

\[ \begin{pmatrix} x \\ y \end{pmatrix} = A'C \]

\[ = \frac{1}{4} \begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 2 \\ 4 \end{pmatrix} \]

\[ = \begin{pmatrix} 5 \\ -117 \end{pmatrix} \] A1A1 N3

METHOD 2

5x + y = 8, \ 6x + 2y = -4

for work towards solving their system

\[ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -117 \end{pmatrix} \] A1A1 N3 [7 marks]

QUESTION 2

(a) \[ P(A) = \frac{1}{11} \] A1 N1

(b) \[ P(B|A) = \frac{2}{10} \] A2 N2

(c) recognising that \[ P(A\cap B) = P(A) \times P(B|A) \] (M1)

correct values

\[ e.g. P(A\cap B) = \frac{1}{11} \times \frac{2}{10} \]

\[ P(A\cap B) = \frac{2}{110} \] A1 N3 [6 marks]

QUESTION 3

evidence of choosing the product rule \[(M1)\]

\[ f'(x) = e^x \times (-\sin x) + \cos x \times e^x (-\cos x - e^x \sin x) \]

substituting \( \pi \)

\[ e.g. f'(0) = e^0 \cos \pi - e^0 \sin \pi, \ e^0 (-1-0), \ -e^0 \]

taking negative reciprocal \[(M1)\]

\[ e.g. \frac{1}{f'(0)} \]

gradient is \[ \frac{1}{e^0} \] A1 N3 [6 marks]

QUESTION 4

(a) \[
\begin{array}{|c|c|}
\hline
\text{Function} & \text{Graph} \\
\hline
\text{displacement} & A \\
\text{acceleration} & B \\
\hline
\end{array}
\]

\[ \text{A2A2 N4} \]

(b) \[ t = 3 \] A2 N2 [6 marks]

QUESTION 5

(a) in any order translated 1 unit to the right \[ \text{AI N1} \]

stretched vertically by factor 2 \[ \text{AI N1} \]

(b) METHOD 1

Finding coordinates of image on \( g \)

\[ e.g. -1 + 1 = 0, 1x - 2 = -1, (1,1) \rightarrow (-1,1), (0,2) \]

\[ P \text{ is } (3,0) \] A1A1 N4

METHOD 2

\[ h(x) = 2(x-4)^2 - 2 \] (A1)(A1)

\[ P \text{ is } (3,0) \] A1A1 N4 [6 marks]
QUESTION 6

(a) (i) interchanging \( x \) and \( y \) (seen anywhere)
\[ e.g. \quad x = e^{y^2} \]
correct manipulation \( A1 \)
\[ e.g. \quad \ln x = y + 3, \quad \ln y = x + 3 \]
\[ f^{-1}(x) = \ln x - 3 \quad AG \]

(ii) \( x > 0 \) \( A1 \)

(b) collecting like terms; using laws of logs \( (AI)(AI) \)
\[ e.g. \quad \ln x - \ln \left( \frac{1}{x} \right) = 3, \quad \ln x + \ln x - 3 = \ln \left( \frac{x}{x^3} \right) = 3, \quad \ln x^2 = 3 \]
simplify \( (AI) \)
\[ e.g. \quad \ln x = \frac{3}{2}, \quad x^2 = e^6 \]
\[ x = e^{\frac{3}{2}} = \sqrt{e^6} \quad A1 \]

QUESTION 7

attempt to substitute into formula \( V = \int y' dx \)
integral expression \( (M1) \)
\[ e.g. \quad \int \left( \sqrt{4} \right) dx, \quad \int x \]
correct integration \( (AI) \)
\[ e.g. \quad \int x dx = \frac{1}{2}x^2 \]
correct substitution \( V = \pi \left[ \frac{1}{2}a^2 \right] \) \( (AI) \)
equating their expression to \( 32\pi \) \( MI \)
\[ e.g. \quad \pi \left[ \frac{1}{2}a^2 \right] = 32\pi \]
\[ a^2 = 64 \quad A2 \]
\[ a = 8 \quad N2 \]

[7 marks]

QUESTION 8

SECTION B

(a) (i) \( x = 3\cos \theta \) \( A1 \)
\[ y = 3\sin \theta \] \( A1 \)

(ii) \( y = 3\sin \theta \) \( A1 \)

(b) finding area \( (M1) \)
\[ e.g. \quad A = 2x \times 2y, \quad A = 8 \times \frac{1}{2} \]
substituting \( A1 \)
\[ e.g. \quad A = 4 \times 3\sin \theta \times 3\cos \theta, \quad A = 12 \times 3\sin \theta \times 3\cos \theta \]
\[ A = 36\sin 2\theta \quad AG \]
\[ A = 18 \times 2\theta \quad A1 \]

[3 marks]

(c) (i) \( \frac{dA}{d\theta} = 36\cos 2\theta \) \( A2 \)

(ii) for setting derivative equal to 0 \( (M1) \)
\[ e.g. \quad 36\cos 2\theta = 0, \quad \frac{dA}{d\theta} = 0 \]
\[ 2\theta = \frac{\pi}{2} \quad (AI) \]
\[ \theta = \frac{\pi}{4} \quad A1 \]

(iii) valid reason (seen anywhere) \( R1 \)
\[ e.g. \quad \frac{dA}{d\theta} \neq 0; \quad \text{maximum when } f''(x) < 0 \]
finding second derivative \( \frac{d^2A}{d\theta^2} = -72\sin 2\theta \) \( A1 \)
evidence of substituting \( \frac{\pi}{4} \) \( MI \)
\[ e.g. \quad -72\sin \left( 2 \times \frac{\pi}{2} \right) = -72\sin \left( \frac{\pi}{2} \right) = -72 \]
\[ \theta = \frac{\pi}{4} \text{ produces the maximum area} \quad AG \]

[8 marks]

Total [13 marks]
QUESTION 9

(a) (i) evidence of approach

\[ \text{e.g. } \mathbf{PQ} = \mathbf{OP} + \mathbf{OQ}, \mathbf{Q} - \mathbf{P} \]\n\[ \mathbf{PQ} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \]
\[ \mathbf{OP} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]
\[ \mathbf{OQ} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \]

(ii) \[ \mathbf{PR} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \]

[3 marks]

(b) METHOD 1

choosing correct vectors \( \mathbf{PQ} \) and \( \mathbf{PR} \)

finding \( \mathbf{PQ}, \mathbf{PR}, \mathbf{PQ} \) and \( \mathbf{PR} \)

\[ \mathbf{PQ} = \sqrt{3^2 + 2^2 + 1^2} = 6, \quad \mathbf{PR} = \sqrt{2^2 + 3^2 + 2^2} = \sqrt{42} \]

substituting into formula for angle between two vectors

\[ \text{e.g. } \cos \theta = \frac{\mathbf{PQ} \cdot \mathbf{PR}}{\|\mathbf{PQ}\| \|\mathbf{PR}\|} = \frac{6 \times \sqrt{42}}{6 \times \sqrt{42}} = \frac{1}{2} \]

simplifying to expression clearly leading to \( \frac{1}{2} \)

\[ \text{e.g. } \frac{6 \times \sqrt{42}}{6 \times \sqrt{42}} = \frac{1}{2} \]

AG N0

METHOD 2

evidence of choosing cosine rule (seen anywhere)

\[ \mathbf{QR} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \]

\[ \mathbf{PQ} = \sqrt{3^2 + 2^2 + 1^2} = 6, \quad \mathbf{PR} = \sqrt{2^2 + 3^2 + 2^2} = \sqrt{42} \]

\[ \cos \theta = \frac{\mathbf{PQ} \cdot \mathbf{PR}}{\|\mathbf{PQ}\| \|\mathbf{PR}\|} = \frac{6 \times \sqrt{42}}{6 \times \sqrt{42}} = \frac{1}{2} \]

[7 marks]

continued …
QUESTION 10

(a) **METHOD 1**
evidence of substituting $-x$ for $x$ \( (M1) \)
\[
f(-x) = \frac{a(-x)}{(-x)^2 + 1}
\]
\[
f(-x) = -\frac{a}{x^2 + 1} (-f(x))
\]

**METHOD 2**
y = \(-f(x)\) is reflection of \(y = f(x)\) in x axis
and \(y = f(-x)\) is reflection of \(y = f(x)\) in y axis
sketch showing these are the same \( (M1) \)
\[
f(-x) = -\frac{a}{x^2 + 1} (-f(x))
\]

(b) evidence of appropriate approach \( (M1) \)
e.g. \( f'(x) = 0 \)
to set the numerator equal to 0 \( (AI) \)
e.g. \[2ax(x^2 - 3) = 0; (x^2 - 3) = 0\]
\[
(0, 0), \left(\sqrt{3}, \frac{\sqrt{3}}{2}\right), \left(-\sqrt{3}, -\frac{\sqrt{3}}{2}\right) \quad \text{(accept } x = 0, y = 0 \text{ etc.)} \]
\[A1A1A1A1A1 \]

[2 marks]

(c) (i) correct expression \( (A2) \)
e.g. \[\frac{a}{2} \ln((x^2 + 1)^2) - \frac{a}{2} \ln((a - x^2) + 10)\]
area = \[\frac{a}{2} \ln 5\]

(ii) **METHOD 1**
recognizing the shift that does not change the area \( (M1) \)
e.g. \[\int_0^a f(x - 1)dx = \int_0^a f(x)dx = \frac{a}{2} \ln 5\]
recognizing that the factor of 2 doubles the area \( (M1) \)
e.g. \[\int_0^a 2f(x - 1)dx = 2\int_0^a f(x - 1)dx = 2\int_0^a f(x)dx\]
\[\int_0^a 2f(x - 1)dx = a \ln 5 \quad \text{(i.e. 2 x their answer to (c)(i))} \]
\[A1 \]

**METHOD 2**
changing variable let \( w = x - 1 \), so \( dw = dx \)
\[2 \int f(w)dw = 2a \ln(w^2 + 1) + c \]
substituting correct limits \( (M1) \)
e.g. \[a \ln((x - 1)^2 + 1) - a \ln((x - 1)^2 + 10)\]
\[\int_0^a 2f(x - 1)dx = a \ln 5 \]
\[A1 \]

[7 marks]

continued ….
SECTION A

Answer all the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Let \( A = \begin{pmatrix} 5 & 1 \\ 6 & 2 \end{pmatrix} \) and \( B = \begin{pmatrix} 2 & -1 \\ 6 & 5 \end{pmatrix} \).

(a) (i) Find \( AB \). [2 marks]

(ii) Write down the inverse of \( A \). [3 marks]

Let \( X = \begin{pmatrix} x \\ y \end{pmatrix} \) and \( C = \begin{pmatrix} 8 \\ -4 \end{pmatrix} \).

(b) Solve the matrix equation \( AX = C \). [4 marks]

2. [Maximum mark: 6]

The letters of the word PROBABILITY are written on 11 cards as shown below.

PROBABILITY

Two cards are drawn at random without replacement. Let \( A \) be the event the first card drawn is the letter A. Let \( B \) be the event the second card drawn is the letter B.

(a) Find \( P(A) \). [1 mark]

(b) Find \( P(B|A) \). [2 marks]

(c) Find \( P(A\cap B) \). [3 marks]
3. [Maximum mark: 6]

Let \( f(x) = e^x \cos x \). Find the gradient of the normal to the curve of \( f \) at \( x = \pi \).

4. [Maximum mark: 6]

The following diagram shows the graphs of the displacement, velocity and acceleration of a moving object as functions of time, \( t \).

(a) Complete the following table by noting which graph A, B or C corresponds to each function.

<table>
<thead>
<tr>
<th>Function</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>displacement</td>
<td></td>
</tr>
<tr>
<td>acceleration</td>
<td></td>
</tr>
</tbody>
</table>

(b) Write down the value of \( t \) when the velocity is greatest.
5. [Maximum mark: 6]
Let \( f(x) = x^3 \) and \( g(x) = 2(x-1)^2 \).
(a) The graph of \( g \) can be obtained from the graph of \( f \) using two transformations. Give a full geometric description of each of the two transformations. [2 marks]
(b) The graph of \( g \) is translated by the vector \( \begin{pmatrix} 3 \\ -2 \end{pmatrix} \) to give the graph of \( h \). The point \((-1, 1)\) on the graph of \( f \) is translated to the point \( P \) on the graph of \( h \). Find the coordinates of \( P \). [4 marks]

6. [Maximum mark: 7]
Let \( f(x) = e^x \).
(a) (i) Show that \( f'(x) = \ln x - 3 \).
(ii) Write down the domain of \( f^{-1} \). [3 marks]
(b) Solve the equation \( f^{-1}(x) = \ln \left( \frac{1}{x} \right) \). [4 marks]
7. (Maximum mark: 7)

The graph of \( y = \sqrt{x} \) between \( x = 0 \) and \( x = a \) is rotated 360° about the x-axis.

The volume of the solid formed is 32π. Find the value of \( a \).

---

Do NOT write on this page.

SECTION B

Answer all the questions on the answer sheets provided. Please start each question on a new page.

8. (Maximum mark: 11)

A rectangle is inscribed in a circle of radius 3 cm and centre \( O \), as shown below.

The point \( P(x, y) \) is a vertex of the rectangle and also lies on the circle. The angle between \( (OP) \) and the x-axis is \( \theta \) radians, where \( 0 \leq \theta \leq \frac{\pi}{2} \).

(a) Write down an expression in terms of \( \theta \) for

(i) \( x \);

(ii) \( y \).

Let the area of the rectangle be \( A \).

(b) Show that \( A = 18 \sin 2\theta \).

(c) (i) Find \( \frac{dA}{d\theta} \).

(ii) Hence, find the exact value of \( \theta \) which maximizes the area of the rectangle.

(iii) Use the second derivative to justify that this value of \( \theta \) does give a maximum.
9. (Maximum mark: 16)

The vertices of the triangle PQR are defined by the position vectors

\[ \vec{OP} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \quad \vec{OQ} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \quad \text{and} \quad \vec{OR} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \]

(a) Find

(i) \( \vec{PQ} \); \quad [3 marks]

(ii) \( \vec{PR} \); \quad [7 marks]

(b) Show that \( \cos \angle RPQ = \frac{1}{2} \).

(c) (i) Find \( \sin \angle RPQ \).

(ii) Hence, find the area of triangle PQR, giving your answer in the form \( \sqrt{3} \). \quad [6 marks]

10. (Maximum mark: 16)

Let \( f(x) = \frac{ax}{x^2 + 1} \), \(-8 \leq x \leq 8, \ a \in \mathbb{R} \). The graph of \( f \) is shown below.

The region between \( x = 3 \) and \( x = 7 \) is shaded.

(a) Show that \( f(-x) = -f(x) \). \quad [2 marks]

(b) Given that \( f''(x) = \frac{2ax(x^2 - 3)}{(x^2 + 1)^3} \), find the coordinates of all points of inflexion. \quad [7 marks]

(c) It is given that \( \int f(x) \, dx = \frac{a}{2} \ln(x^2 + 1) + C \).

(i) Find the area of the shaded region, giving your answer in the form \( a \ln 4 \). \quad [7 marks]

(ii) Find the value of \( \int_{-8}^{8} f(x) \, dx \). \quad [7 marks]
SECTION A

QUESTION 1

(a) for interchanging \( x \) and \( y \) (may be done later)
\[
g^{-1}(x) = \frac{x + 3}{2} \quad \text{(AI)}
\]
\[
(accept \quad y = \frac{x + 3}{2}, \quad x = \frac{y + 3}{2}) \quad \text{(M1)}
\]
\( \quad \text{A1} \quad \text{N2} \)

(b) METHOD 1
\[
g(4) = 5
\]
\( \quad \text{(AI)} \)
\( \quad \text{A1} \quad \text{N3} \)

METHOD 2
\[
f \cdot g(4) = (2x - 3)^2
\]
\( f \cdot g(4) = (2 \cdot 4 - 3)^2 \quad \text{(AI)}
\]
\( = 25 \quad \text{(A1)} \)
\( \quad \text{N3} \quad \text{[5 marks]} \)

QUESTION 2

finding scalar product and magnitudes
\( \quad \text{(AI)(A1)(A1)} \)
scalar product \( = 12 - 20 - 15 \quad (= -23) \)
magnitudes \( = \sqrt{5^2 + 4^2 + 5^2} \quad \text{and} \quad \sqrt{(-5)^2 + (-3)^2} \quad (\sqrt{50}, \sqrt{34}) \)
substitution into formula
\( \quad e.g. \quad \cos \theta = \frac{\sqrt{5^2 + 4^2 + 5^2} \cdot \sqrt{(-5)^2 + (-3)^2}}{\sqrt{50} \cdot \sqrt{34}} \quad \text{(M1)}
\]
\( \quad \cos \theta = \frac{-23}{50} \quad (= -0.46) \quad \text{(A2)} \)
\( \quad \text{A2} \quad \text{N4} \quad \text{[6 marks]} \)

QUESTION 3

(a) \( a = 10 \quad \text{(AI)} \)
\( \quad \text{A1} \quad \text{N1} \)

(b) \( a = p, \; b = 2q \) (or \( a = 2q, \; b = p \))
\( \quad \text{A1A1} \quad \text{NIN1} \)

(c) \( \frac{10}{5} \quad p = (2q)^\frac{1}{3} \quad \text{A1A1A1} \quad \text{N3} \quad \text{[6 marks]} \)

QUESTION 4

(a) \( 5 \quad \text{A1} \quad \text{N1} \)

(b) METHOD 1
\[
\log \frac{32^x}{8} - \log_2 32^x - \log_8 8
\]
\( \quad \log \frac{32^x}{8} \quad = x \log_2 32 - y \log_8 8 \quad \text{(AI)}
\]
\( \quad \log_8 8 = 3 \quad \text{(A1)}
\]
\( p = 5, \; q = -3 \quad \text{(accept } 5x - 3y) \quad \text{(A1)} \)

METHOD 2
\[
\frac{32^x}{8} \quad \left( \frac{32^y}{8} \right)^2
\]
\( = \frac{2^n}{2^m} = 2^{n-2m} \quad \text{(AI)}
\]
\( \log_2 (2^{n-2m} - 5x - 3y) \quad \text{(A1)}
\]
\( p = 5, \; q = -3 \quad \text{(accept } 5x - 3y) \quad \text{(A1)} \)
\( \quad \text{A1} \quad \text{N3} \quad \text{[5 marks]} \)
QUESTION 5

(a) METHOD 1

\[ M = (M^{-1})^T \]

\[ M = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} \]

METHOD 2

\[ \begin{pmatrix} a & h \\ b & d \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix} \]

\[ a + b = 1, \quad 2b = 0, \quad 5c + d = 0, \quad 2d = 1 \]

\[ M = \begin{pmatrix} 0.2 & 0 \\ -0.1 & 0.5 \end{pmatrix} \]

(b) METHOD 1

Evidence of an appropriate approach

e.g. \[ X = MB \]

\[ \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \]

\[ x = 15 \]

METHOD 2

Evidence of an appropriate approach

e.g. \[ \begin{pmatrix} 0.2 & 0 \\ -0.1 & 0.5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \]

\[ 0.2x + 1, \quad -0.1x + 0.5y = 7 \]

\[ x = 5, \quad y = 15 \]

[6 marks]

QUESTION 6

(a) METHOD 1

\[ f''(x) = 3(x - 3)^2 \]

METHOD 2

Attempt to expand \((x - 3)^2\)

e.g. \[ f'(x) = x^2 - 9x + 27 \]

\[ f''(x) = 3x^2 - 18x + 27 \]

METHOD 3

Finding \( f(x) = \frac{1}{4}(x - 3)^2 + c \) and sketching this function

Indicating minimum at \( x = 3 \)

[3 marks]
QUESTION 7

\[ e^{x} \left( \sqrt{3} \sin x + \cos x \right) = 0 \]

\[ e^{x} = 0 \] not possible (seen elsewhere)

simplifying

\[ e.g. \quad \sqrt{3} \sin x + \cos x = 0, \quad \sqrt{3} \sin x = -\cos x, \quad \frac{\sin x}{\cos x} = -\frac{1}{\sqrt{3}} \]

EITHER

\[ \tan x = -\frac{1}{\sqrt{3}} \]

\[ x = \frac{5\pi}{6} \]

A1

A2 N4

OR

sketch of 30°, 60°, 90° triangle with sides 1, 2, \( \sqrt{3} \)

work leading to \( x = \frac{5\pi}{6} \)

A1

A1

verifying \( \frac{5\pi}{6} \) satisfies equation

A1 N4

[6 marks]

QUESTION 8

(a) (i) \( -3e^{-x} \)

(ii) \( \cos \left( x - \frac{\pi}{3} \right) \)

A1 N1

(b) evidence of choosing product rule

\[ e.g. \quad u^2 \cdot v' \]

correct expression

\[ e.g. \quad -3e^{-x} \sin \left( x - \frac{\pi}{3} \right) + e^{-x} \cos \left( x - \frac{\pi}{3} \right) \]

complete correct substitution of \( x = \frac{\pi}{3} \)

\[ e.g. \quad -3e^{-\frac{\pi}{3}} \sin \left( \frac{\pi}{3} - \frac{\pi}{3} \right) + e^{-\frac{\pi}{3}} \cos \left( \frac{\pi}{3} - \frac{\pi}{3} \right) \]

A1 N3

[6 marks]
SECTION B

QUESTION 9

(a) 
<table>
<thead>
<tr>
<th>3, 9</th>
<th>4, 9</th>
<th>5, 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 10</td>
<td>4, 10</td>
<td>5, 10</td>
</tr>
</tbody>
</table>

(b) 12, 13, 14, 15 (accept 12, 13, 14, 15, 16, 17, 18, 19, 20)

(c) P(12) = \frac{1}{9}, P(13) = \frac{3}{9}, P(14) = \frac{3}{9}, P(15) = \frac{2}{9}

(d) correct substitution into formula for \( E(X) \)
\[ E(S) = 12 \cdot \frac{1}{9} + 13 \cdot \frac{2}{9} + 14 \cdot \frac{3}{9} + 15 \cdot \frac{2}{9} \]
\[ E(S) = 13 \frac{1}{9} \]

(e) METHOD 1
- correct expression for expected gain \( E(A) \) for 1 game
\[ E(A) = \frac{4}{9} \cdot 50 - \frac{5}{9} \cdot 30 \]
\[ E(A) = \frac{20}{9} \]
- amount at end = expected gain for 1 game \times 36
\[ = 200 \text{ (dollars)} \]

METHOD 2
- attempt to find expected number of wins and losses
\[ E = \frac{4}{9} \cdot 36, \frac{5}{9} \cdot 36 \]
- attempt to find expected gain \( E(G) \)
\[ E(G) = 200 \] (dollars)

A2 N2 [12 marks]

QUESTION 10

(a) \[ L = \begin{bmatrix} 8 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \]

(b) evidence of equating \( r \) and \( \tilde{oA} \)
\[ e.g. \begin{bmatrix} 6 \\ -2 \\
-1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix} - r \]
- one correct equation
\[ \begin{align*}
L &= 2 \cdot 2r \\
&= 4 - 2s \\
&= -1 + 5s
\end{align*} \]
\[ s = 2 \]
- evidence of confirming for other two equations
\[ e.g. 6 = 2 + 4, 2 = 4 - 2, 9 = -1 + 10 \]
so A has on \( z \).

A3 N1 [12 marks]

continued...
Question 11 continued

(c) (i) evidence of approach

\( e.g. \begin{pmatrix} 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \)

\( \begin{pmatrix} 5 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \)

one correct equation

\( e.g. 2x + 2y = 8, 4x + 3y = 1, -1 + 5z = t \)

attempt to solve

\( M1 \)

finding \( x = 3 \)

substituting

\( e.g. \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix} \)

\( M1 \)

\( A1 \)

(ii) evidence of appropriate approach

\( e.g. AB = AO + OB, AB = OB - OA \)

\( \begin{pmatrix} 2 \\ 5 \end{pmatrix} \)

\( A1 \)

\( N2 \)

Total \( 16 \) marks

(d) evidence of appropriate approach

\( e.g. AB = DC \)

correct values

\( e.g. \begin{pmatrix} 2 \\ 5 \\ -4 \\ -5 \end{pmatrix} \)

\( \begin{pmatrix} 2 \\ 5 \\ -4 \\ -5 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \\ -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ -4 \\ -5 \end{pmatrix} \)

\( A1 \)

\( N2 \)

\( \{3 \text{ marks}\} \)

QUESTION 11

Note: In this question, do not penalize absence of units.

(a) (i) \( s = \int \frac{40 - at}{a} \, dt \)

\( M1 \)

\( s = \frac{40t}{2} - \frac{at^2}{2} + c \)

\( A1(\, A1) \)

substituting \( t = 0 \) when \( c = 100 \)

\( M1 \)

\( s = 40 \frac{t^2}{2} + 100 \)

\( A1 \)

\( N5 \)

(ii) \( s = 40 \frac{t^2}{2} \)

\( A1 \)

\( N1 \)

\( \{6 \text{ marks}\} \)

(b) (i) stops at station, so \( v = 0 \)

\( t = \frac{40}{a} \) (seconds)

\( M1 \)

\( A1 \)

\( N2 \)

(ii) evidence of choosing formula for \( s \) from (a) (ii)

\( M1 \)

\( \text{substituting } t = \frac{40}{a} \)

\( e.g. 40 \frac{40}{a} - \frac{1}{2} \frac{a}{2} 40 \frac{40}{a} \)

\( A1 \)

\( N5 \)

setting up equation

\( e.g. 500 = a; 500 = 40 - \frac{a}{2} + \frac{40}{2} \)

\( M1 \)

\( 500 = 800 - \frac{1000a}{a} \)

\( M1 \)

\( \text{evidence of simplification to an expression which obviously leads to } a = \frac{8}{5} \)

\( e.g. 500a = 800, 5 = \frac{8}{5} \)

\( a = \frac{8}{5} \)

\( A1 \)

\( N6 \)

\( \{6 \text{ marks}\} \)

continued...
Question 11 continued

(c) METHOD 1
v = 40 - 4t
40 - 4t = 0
4t = 40
\( t = 10 \) (A1)

substituting into expression for \( s \)
\( s = 40 \times 10 \times \frac{1}{2} \times 4 \times 10^2 \)
s = 200
since 200 < 500 (allow FT on their s, if \( s < 500 \))

METHOD 2
from (b) \( t = \frac{40}{4} = 10 \) (A2)

substituting into expression for \( s \)
e.g. \( s = 40 \times 10 \times \frac{1}{2} \times 4 \times 10^2 \)
s = 200
since 200 < 500, train stops before the station

METHOD 3
\( a \) is deceleration
\( 4 < \frac{8}{5} \) (A1)

so stops in shorter time
so less distance travelled
so stops before station

Total [17 marks]
QUESTION 4

(a) METHOD 1

- evidence of choosing the cosine formula
  - correct substitution
  \[ \cos \angle ABC = \frac{a^2 + c^2 - b^2}{2ac} \]
  \[ \angle ABC = 2.38 \text{ radians} \approx (136^\circ) \]

- METHOD 2

- evidence of appropriate approach involving right-angled triangles
  - correct substitution
  \[ \sin \left( \frac{\angle ABC}{2} \right) = \frac{6.5}{7} \]
  \[ \angle ABC = 2.38 \text{ radians} \approx (136^\circ) \]

(b) METHOD 1

- \( \angle ACD = \pi - 2.38 \) \((180 - 136.4)\)
  - evidence of choosing the sine rule in triangle ACD
  - correct substitution
  \[ \sin \theta = \frac{6.5}{7} \]
  \[ \theta = \sin^{-1} \left( \frac{6.5}{7} \right) \approx 61.54^\circ \]

- METHOD 2

- \( \angle ABD = 2.38 \) \((180 - 136.4)\)
  - evidence of choosing the sine rule in triangle ABD
  - correct substitution
  \[ \sin \theta = \frac{6.5}{7} \]
  \[ \theta = \sin^{-1} \left( \frac{6.5}{7} \right) \approx 61.54^\circ \]

Note: Two triangles are possible with the given information. If candidate finds \( \angle ADC = 2.31 \) \((132^\circ) \) leading to \( \angle CAD = 0.076 \) \((4.35^\circ) \), award marks as per markscheme.

QUESTION 5

(a) \[ S_n = \sum_{i=1}^{n} a_i = 2^n + 3^n + 4^n + 5^n \] (accept 56 + 32 + 64 + 128)

(b) (i) METHOD 1

- recognizing a GP
  - correct substitution into formula for sum
  \[ S_n = \frac{2(2^n - 1)}{2-1} \]
  \[ S_{10} = 2147483632 \]

(ii) valid reason (e.g. infinite GP, diverging series, \( |r| > 1 \))

- METHOD 2

- recognizing \( \sum_{k=1}^{n} \sum_{j=1}^{m} a_{ij} = \sum_{k=1}^{n} (2k + 1) \)
  - correct substitution into formula for sum
  \[ S_n = \frac{(2(2^n - 1))}{2-1} \]
  \[ S_{10} = 2147483646 \]
  \[ \sum_{k=1}^{10} = 2147483646 - (2 + 4 + 8) \]
  \[ = 2147483632 \]

- valid reason (e.g. infinite GP, diverging series, \( |r| > 1 \))

[7 marks]
QUESTION 6
(a) gradient is 0.6
(b) at R, y = 0 (seen anywhere)
   at x = 2, y = ln5 (=1.609...)
   gradient of normal = -1.666...
   evidence of finding correct equation of normal
   e.g. \(\frac{y - 2}{x - 2} = \frac{1}{-1.609}\), \(y = -1.67x + c\)
   \(x = 2.97\) (correct 2.96)
   coordinates of R are (2.97, 0)

QUESTION 7
(a) attempt to use discriminant
   correct substitution, \((k - 3)^2 - 8 + 1\)
   setting their discriminant equal to zero
   e.g. \((k - 3)^2 - 4k - 2 = 0\)
   \(k = 3, k = -8\)

(b) \(k = 1, k = 9\)

SECTION B
QUESTION 8
(a) finding the limits \(x = 0, x = 5\)
   integral expression
   e.g. \(\int f(x)dx\)
   area = 52.1

(b) evidence of using formula \(v = \int y^2dx\)
   correct expression
   e.g. volume = \(\pi \int \int (x-5)^2dx\)
   volume = 2540

(c) area is \(\int (a-x)dx\)
   e.g. \(\frac{a - \sqrt{a^2 - \frac{a^2}{3}}}{2}\)
   substituting limits
   e.g. \(\frac{3}{2}\)
   setting expression equal to area of R
   e.g. \(a = 52.1, a' = 68\times52.1\)
   \(a = 6.79\)
QUESTION 9

A = N(46, 10²) B = N(µ, 12²)
(a) \( P(Z > 60) = 0.0008 \) [2 marks]

(b) correct approach
\( \frac{Z < \frac{60 - 46}{\sqrt{12}}}{\frac{60 - 46}{12}} = 0.05, \text{ sketch} \)
\( \mu = 47.6 \) [3 marks]

(c) (i) reject A
(ii) METHOD 1
\( P(Z > 60) = 1 - 0.9992 = 0.0008 \)
valid reason
\( \text{e.g. probability of A getting there on time is greater than probability of B} \)
\( 0.9992 > 0.85 \) [3 marks]

METHOD 2
\( P(Z > 60) = 1 - 0.85 = 0.15 \)
valid reason
\( \text{e.g. probability of A getting there late is less than probability of B} \)
\( 0.0008 < 0.15 \) [3 marks]

(d) (i) Let \( X \) be the number of days when the van arrives before 9:00
\( P(X = 5) = 0.855 \)
\( = 0.444 \) [3 marks]

(ii) METHOD 1
\( \text{evidence of adding correct probabilities} \)
\( \text{e.g. } P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5) \)
correct values \( 0.1382 + 0.3915 = 0.4437 \)
\( P(X \geq 3) = 0.973 \) [3 marks]

METHOD 2
\( \text{evidence of using the complement} \)
\( \text{e.g. } P(X \geq 3) = 1 - P(X < 2) = 1 - 0.0266 \)
correct values \( 1 - 0.0266 = 0.973 \) [3 marks]

Total [13 marks]

---

QUESTION 10

\( f(t) = 5 \sin(\pi t + 0.927) \) (accept \( p = 5, q = 1, r = 0.927 \) ) [3 marks]

(b) (i) 5
(ii) \( 2\pi (0.28) \)
(iii) -0.927
(A1A1A1) [3 marks]

(c) evidence of correct approach
\( \text{e.g. max/min, sketch of } f'(t) \text{ indicating noon} \)

\( f(t) \) [3 marks]

(d) evidence of correct approach
\( \text{e.g. max/min, sketch of } f''(t) \text{ indicating noon} \)

\( f''(t) \) [3 marks]

Note: Award AI for approximately sinusoidal shape,
AI for end points approximately correct, \((-2\pi, 4), (2\pi, 4)\)
AI for approximately correct position of graph, \((y\text{-intercept } (0, 4), \text{ maximum to right of } y\text{-axis})\)
Question 10 continued

(e) \( k = -5, k = 5 \)

(f) **METHOD 1**

graphical approach (but must involve derivative functions)

\[ y = \frac{1}{x^3} \]

\[ f'(x) = -3x^2 \cdot 4 \sin(x) \]

Each curve

\( x = 0.511 \)

**METHOD 2**

\[ g'(x) = \frac{1}{x^4} \]

\[ f'(x) = 3 \cos(x) - 4 \sin(x) \]

Evidence of attempt to solve \( g'(x) = f'(x) \)

\( x = 0.511 \)

\[ M1 \]

\[ M1 \]

\[ M1 \]

\[ M1 \]

\[ A2 \]

\[ A2 \]

\[ A2 \]

\[ A2 \]
QUESTION 2

(a) METHOD 1
choosing cosine rule
substituting correctly
\[ \cos \alpha = \frac{3.9^2 + 5.2^2 - 2(3.9)(5.2) \cos \beta}{2(3.9)(5.2)} \]
\[ \alpha = 61.1 \text{ (cm)} \]

METHODOLOGY

 evidence of approach involving right-angled triangles
substituting correctly
\[ \sin 0.9 = \frac{1}{3.9} \]
\[ AB = 3.9 \sin 0.9 \]
\[ AB = 6.11 \text{ (cm)} \]

METHOD 3
choosing the sine rule
substituting correctly
\[ \sin \theta /1.8 = \sin 1.8 /3.9 \]
\[ AB = 6.11 \text{ (cm)} \]

(b) METHOD 1
reflex \( \angle AOB = 2\theta = 1.8 \) (\( \approx 4.4832 \))
correct substitution
\[ A = \frac{1}{2} \times 3.9 \times \angle (4.4832...) \]
area \( A = 34.1 \text{ (cm}^2) \)

METHOD 2
finding area of circle \( A = \pi (3.9)^2 \approx 47.78... \)
finding area of (minor) sector \( A = \frac{1}{2} (3.9)^2 \times 1.8 \approx 13.68... \)
subtracting
\[ \angle (3.9)^2 - \frac{1}{2} (3.9)^2 \times 1.8 \approx 34.1 \text{ (cm}^2) \]

METHOD 3
finding reflex \( \angle AOB = 2\theta = 1.8 \) (\( \approx 4.4832 \))
finding proportion of total area of circle
\[ \frac{2\pi - 1.8}{2\pi} \times \pi (3.9)^2 = \frac{(3.9)^2 \times 2}{2\pi} \]
area \( A = 34.1 \text{ (cm}^2) \)

QUESTION 3

(a) attempt to form any composition (even if order is reversed)
correct composition \( h(x) = g \left( \frac{3x + 1}{2} \right) \)
\[ h(x) = 4 \cos \left( \frac{3x + 1}{2} - 1 \right) \]
\[ 4 \cos \left( \frac{3x}{2} + 1 - 1 \right) \]
\[ 4 \cos \left( \frac{3x}{2} + 2 - 1 \right) \]
\[ A1 \]

(b) period is \( 4\pi /12, 6 \)
\[ A1 \]

(c) range is \(-5 \leq h(x) < 3\) \((-5, 3)\)
\[ A1A1 \]

[6 marks]

QUESTION 4

(a) evidence of attempt to find \( P(X \leq 475) \)
e.g. \( P(X \leq 5.3) \approx 0.994 \)
\[ A1 \]

(b) evidence of using the complement
e.g. \( 0.73, 1 - \rho \)
\[ A1 \]

setting up equation
e.g. \( \frac{a}{20} = 0.6128 \)
\[ a = 462 \]
\[ A1 \]

[6 marks]

QUESTION 5

evidence of appropriate approach
e.g. \[ \begin{pmatrix} 2 & 3 & 5 \\ -1 & 2 & 5 \\ 1 & 2 & -1 \end{pmatrix} \]
two correct equations
e.g. \[ 2 + 5r = 9 - 3r, 3 - 3w = 2 + 5r \]
\[ -1 + 2x = 2 - t \]
attempting to solve the equations
one correct parameter \( r = 2, t = -1 \)
\[ A1A1 \]

[5 marks]
QUESTION 6

(a) evidence of substituting into formula for $a^n$ term of GP
\[
\frac{1}{81} = \frac{1}{3}r^n
\]
setting up correct equation \[
\frac{1}{81} = \frac{1}{3}r^n
\]
\[
r = 3
\]

(M1)

A1

A1 N2

(b) METHOD 1
setting up an inequality (accept an equation)
\[
\frac{1}{3}(3^n - 1) > 40; \quad \frac{1}{3}(3^n - 1) > 40; \quad 3^n > 6481
\]
evidence of solving
\[
n = 7.9888..., 
\]
\[
\therefore n = 8
\]

(M1)

M1

A1

A1 N2

METHOD 2
if $n = 7$, sum = 13.49...; if $n = 8$, sum = 40.49...

$n = 8$ (is the smallest value)

A2

A2 N2

[7 marks]

QUESTION 7

(a) appropriate approach
e.g. tree diagram or a table

\[
P(\text{win}) = P(H\cap W) + P(A\cap W)
\]
\[
= (0.65)(0.83) + (0.35)(0.25)
\]
\[
= 0.5305 \text{ or } 0.531
\]

(M1)

A1

A1 N2

(b) evidence of using complement
e.g. $1 - p$, 0.3695

choosing a formula for conditional probability
e.g. \[ P(H|W') = \frac{P(H\cap W')}{P(W')}. \]
correct substitution
e.g. \[
\frac{0.3695}{0.3695} = \frac{0.1105}{0.3695}
\]
\[
P(\text{home}) = 0.299
\]

(M1)

A1

A1 N2

[8 marks]
SECTION B

QUESTION 9

(a) evidence of substituting (−4, 3)
correct substitution \( 3 = a(-4)^2 + b(-4) + c \)
\[ a \rightarrow \text{N} \] \[ b \rightarrow \text{N} \]

(b) \( 3 = 36a + 6b + c \), \( -1 = 4a - 28 + c \)
\[ \text{A1} \text{N1} \]

(c) (i) \[ A = \begin{pmatrix} 16 & -4 & 1 \\ 36 & 6 & -1 \\ 4 & -2 & 1 \end{pmatrix} \]

(ii) \[ A^{-1} = \begin{pmatrix} 0.05 & 0.0125 & 0.00625 \\ 1 & 0.1 & 0.125 \\ 0.6 & 0.1 & 1.5 \end{pmatrix} \]

(iii) evidence of appropriate method
\[ e.g. \ X = AB \], attempting to solve a system of three equations
\[ X = \begin{pmatrix} 0.25 \\ -0.5 \\ -3 \end{pmatrix} \] (accept fractions)
\[ 0.25 \times x^2 - 0.5x - 3 \] (accept \( a = 0.25, b = -0.5, c = -3 \), or fractions)

(d) \[ f(x) = 0.25(x - 1)^2 - 3.25 \] (accept \( k = 1, a = -3.25, a = 0.25 \), or fractions)

\[ \text{A1} \text{A1} \text{N1} \]

Total [15 marks]
QUESTION 10

(a) attempt to expand
\((x + h)^2 = x^2 + \frac{3}{2}xh + \frac{3}{2}h^2 + h^2\)

(b) evidence of substituting \(x = h\)
correct substitution
\(e.g. \ f'(x) = \lim_{h \to 0} \frac{(x + h)^2 - 4(x + h) + 1 - (x^2 - 4x + 1)}{h}\)
simplifying
\(e.g. \ (x^2 + \frac{3}{2}xh + \frac{3}{2}h^2 + h^2 - 4x - 4h + 1 - x^2 + 4x - 1)\)
factoring out \(h\)
\(\frac{h(3x^2 + 3h + h^2 - 4)}{h}\)
\(f'(x) = 3x^2 - 4\)

(c) \(f'(1) = -1\)
setting up an appropriate equation
e.g. \(3(1)^2 - 4 = -1\)
at \(Q, x = -1, y = 4\) \(Q \in (-1, 4)\)

(d) recognizing that \(f\) is decreasing when \(f'(x) < 0\)
correct values for \(p\) and \(q\) (but do not accept \(p = 1.15, q = -1.15\))
e.g. \(p = -1.15, q = 1.15, \frac{2}{7}\) for an interval such as \(-1.15 < x < 1.15\)

(e) \(f'(x) \leq -4, y \geq -4, [-4, \infty]\)

\(\text{Total [15 marks]}\)